Thermodynamics of holographic models for QCD in the Veneziano limit

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Veneziano QCD

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The holographic model and its vacuum structure described in arXiv:1112.1261, and in the previous talk by Järvinen. For studying the thermodynamics, we add a black hole to the bulk.

Action

To recap the setup, the gravity action is

$$S = M^3 N_c^2 \int d^5x \, \mathcal{L} \equiv \frac{1}{16\pi G_5} \int d^5x \, \mathcal{L}, \tag{1}$$

where

$$\mathcal{L} = \left[\sqrt{-g} \left(R - \frac{4}{3} \frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda) \right) \right. \tag{2}$$
$$\left. - V_f(\lambda,\tau) \sqrt{\det \left(g_{ab} + \kappa(\lambda,\tau)(D_a T)^*(D_b T) + \omega(\lambda,\tau) F_{ab} \right)} \right].$$

The metric Ansatz is

$$ds^2 = b^2(r) \left[-f(r)dt^2 + d\mathbf{x}^2 + \frac{dr^2}{f(r)} \right], \tag{3}$$

and the two scalar functions, $1/\lambda$ sourcing F^2 and τ sourcing $\langle \bar{q}q \rangle$, are

$$\lambda = \lambda(r) = e^{\phi(r)}, \quad \tau = \tau(r), \text{where } T = \tau \mathbb{1}. \tag{4}$$

Finite chemical potential

Field theory chemical potential is dual to the bulk $U(1)$ gauge field A_μ.

- Matter at rest $\Rightarrow A_i = 0$ for $i = 1, 2, 3$
- Gauge transformation $\Rightarrow A_5 = 0$, and τ stays real
- Only A_0 is left.
- e.o.m. for A_0 is cyclic
- Solving the eo.m., A_0 is expressed in terms of the other fields:

$$A_0'(r) = -\frac{b^2}{\mathcal{L}_A^2 \kappa} \sqrt{\left(1 + \frac{f\kappa}{b^2}\dot{\tau}^2\right) \frac{\tilde{n}_1^2}{\tilde{n}_1^2 + (b^3 \kappa V_f)^2}}. \tag{5}$$

Boundary conditions

Gravity equations solved numerically, with boundary conditions set at the horizon:

- Gauge field $A_\mu(r_h) = 0$, $A'_\mu(r_h)$ sets n and μ. This is replaced with the integration constant $\tilde{n}_1$ from the previous slide.

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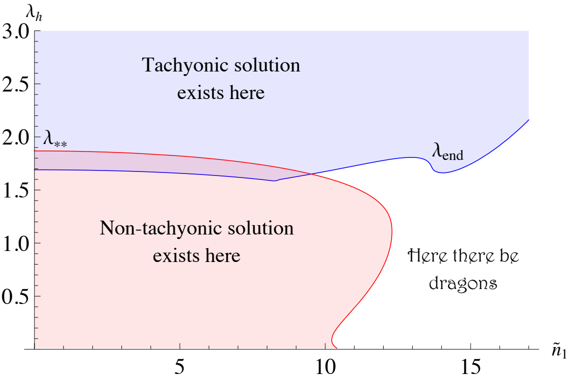
Want $m_q = 0$:

- chiral symmetry intact $\Rightarrow \tau(r) \equiv 0$
- chiral symmetry broken $\Rightarrow \tau(r) \neq 0$, but $m_q = 0$ determines τ_h as a function of $(\tilde{n}_1, \lambda_h)$

Structure of the solution space, pt. 1



Solutions in both branches exist only in certain regions of the $(\tilde{n}_1, \lambda_h)$ -space.

- λ_{**} bounds the region where the tachyon free solution exists
- λ_{end} bounds the region where the tachyonic solution exists.

Thermodynamics

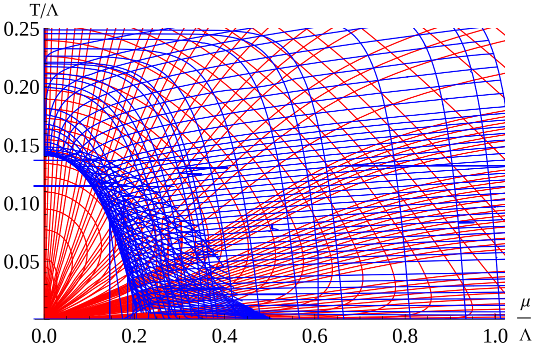
Any solution to the Einstein equations, corresponding to a pair $(\tilde{n}_1, \lambda_h)$ and a choice of tachyon or no tachyon, gives

$$\begin{aligned} T &= -\tfrac{1}{4\pi} f'(r_h; \tilde{n}_1, \lambda_h) & s &= \frac{1}{4G_5} b^3(\tilde{n}_1, \lambda_h) \\ \mu &= \lim_{r\to 0} A_0(r; \tilde{n}_1, \lambda_h) & n &= \frac{1}{4\pi \Lambda^3(\tilde{n}_1, \lambda_h)} \tilde{n}_1 . \end{aligned} \quad (6)$$

Structure of the solution space, pt. 2

Mapping the $(\tilde{n}_1, \lambda_h)$ plane to the (μ, T) -plane we find that given a point (μ, T), we have the candidate solutions

- tachyonic vacuum, which can be compactified to any μ, T
- non-tachyonic black hole, corresponding to a pair $(\tilde{n}_1, \lambda_h)$
- 0, 1 or 2 tachyonic black holes each corresponding to different $(\tilde{n}_1, \lambda_h)$



Computing pressure, $\mu = 0$

Pressure from thermodynamics,

$$dp = s\,dT + n\,d\mu. \tag{7}$$

We need to fix the integration constants in the two branches. Consider first the case $\tilde{n}_1 = 0$ (i.e. $\mu = 0$):

- At the $\lambda_h \to \infty$ limit, the $\tau \neq 0$ solution becomes the $\tau \neq 0$ vacuum solution, i.e. $p_b(\lambda_h = \infty) = 0$.
- At λ_{**}, the $\tau \equiv 0$ solution has $T = 0 \Leftrightarrow$ the $\tau \equiv 0$ vacuum solution.
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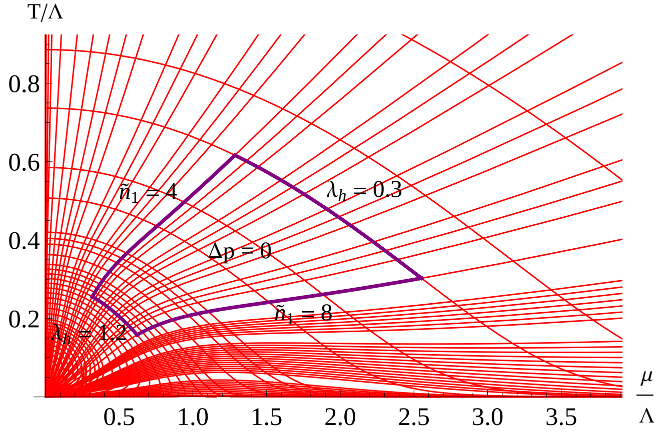
$\Rightarrow$ numerically equivalent to requiring that $p_s(\lambda_{\text{end}}) = p_b(\lambda_{\text{end}})$!

Path independence of pressure

Pressure from

$$dp = s dT + n d\mu$$

must be path independent. This is indeed verified numerically, which is a very non-trivial check of the model and our numerics.



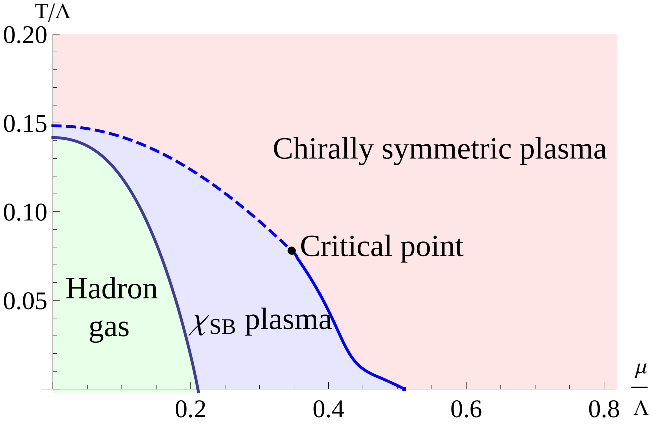
Computing pressure, $\mu > 0$

Given that integrating dp is path independent, we can now fix pressure on the whole (μ, T) -plane:

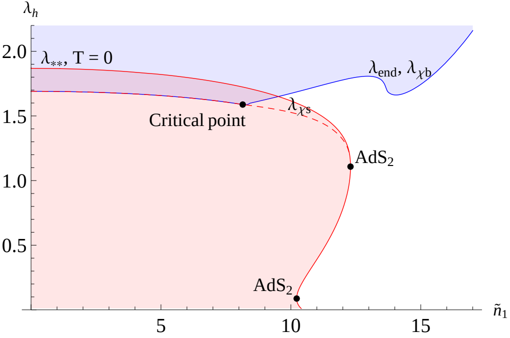
- $p(\lambda_h, \tilde{n}_1 = 0)$ is known for both branches, so first we can integrate along lines of constant λ_h, starting from $\tilde{n}_1 = 0$, and define the pressure along these curves
- then the pressure is defined along $\tilde{n}_1 =$ const. lines by integrating from any intersection with a $\lambda_h =$ const. -line. Multiple intersections give the same constant due to path independence. In practice we use that as a running check on the sensibility of the numerics.
- the above is done separately for the chirally symmetric branch and non-symmetric branch.

Phase diagram, $x_f = 1$

Since we can calculate the pressure, we can determine the stable phase at any (μ, T). This gives us the phase diagram:



How do we end up with the above:



- $T = 0$ curve maps to the $T = 0$ line on the (μ, T) -plane.
- $\lambda_{\chi s}$, chiral transition on the symmetric phase. The critical point is where $\lambda_{\chi s}$ and $\lambda_{\chi b}$ separate.
- $\lambda_{\chi b}$, chiral transition on the non-symmetric phase. Turns out it's the same as $\lambda_{\rm end}$.
- at λ_H, $p_b(\lambda_H) = 0$, is the deconfinement transition (out of the picture)

Outlook

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- Constraining the potentials by matching to QCD
- Moving towards the current best candidate potential

That's all, folks! Thank you!