Predictions from Eternal Inflation

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Over the last several years, a group of people on the fringes of string theory have been working on the idea:

\[
\text{String Theory} \downarrow \\
\text{Eternal Inflation} \downarrow \\
\text{Predictions}
\]

A good time to summarize: some successes, but also shortcomings. Need for new ideas.
Can string theory make some prediction for SUSY breaking scale, nongaussiananities in CMB, ...?
If you have an idea, get in touch with me, because this is my job.

My future employment requires making a connection between string theory and experiment, observation.

If you have an idea, come give a GRAPPA seminar.

My job is not a legitimate string theory job; I did not take a job from anyone in this room. But we do have a string theory job opening at any level.
Observe positive vacuum energy.

String theory $\rightarrow$ Landscape of metastable 4d vacua with $\Lambda \leq 0$, $\Lambda > 0$.

Simplest explanation: we are in a vacuum with $\Lambda > 0$.

One vacuum with $\Gamma < H^4 \rightarrow$ Eternal Inflation.

- Finite initial conditions $\rightarrow$ Infinite spacetime volume in false vacuum.
- False vacuum can decay locally to any other place in landscape.
  
  [Brown Dahlen 11]
- Landscape populated dynamically
Vacua realized as “pocket universes” in “multiverse.”
Q: How to make predictions if string theory has such a rich landscape?

Proposal: Make predictions by counting relative abundance of different events in eternal inflation,

\[
\frac{p_A}{p_B} = \frac{N_A}{N_B}
\]

To compute probability of finding SUSY at LHC, count LHC’s in multiverse.
Good news: \( \lim_{t \to \infty} \frac{N_A(t)}{N_B(t)} \) independent of initial conditions.

\( N_A(t) \): Number of events of type A before time \( t \).

Bad news: depends on choice of \( t \).

Semiclassically: theory is diffeomorphism invariant, no preferred \( t \), \( \frac{N_A}{N_B} \) not well defined.

Computing global \( \Psi \) perturbatively does not solve the problem.
So why persist?

\[ N_A = N_B = \infty \] because counting events in entire spacetime.

Black holes: Semiclassical approximation unreliable when applied to more than one causal patch.
When restrict to one causal patch, infinities disappear.

One approach [Bousso 2006]: Restrict to one causal patch.
Eliminates infinities; also eliminates attractor behavior.
Can we regulate infinities while retaining attractor behavior?

Right thing to do: compute \( \frac{N_A}{N_B} \) in string theory, derive correct regulator.
Impossible right now.
Hope: Whatever correct cutoff (measure) is, it reduces to something simple in semiclassical limit.

**Program:** Guess a measure, and compare to observation. Geometric cutoffs.

**Measure problem:**
Input: Semiclassical spacetime, or ensemble of spacetimes.

Output: \( \frac{N_A}{N_B} \) for any events \( A \) and \( B \).

Usually focus on single semiclassical 4d spacetime. Various corrections do not substantially change the problem.
Over the last several years, a number of well-motivated proposals have been made.

[Linde Linde Mezhlumian 1993]
[Garriga Schwarz-Perlov Vilenkin Winitzki 2005]
[De Simone Guth Salem Vilenkin 2008]
[Bousso 2006] [Bousso 2009] [Bousso Yang 2009]

Several rejected by comparison to observation.

Surprising dualities between apparently very different proposals.

→ A few viable proposals at this time.
My work culminated in 4 papers [Bousso BF Leichenauer Rosenhaus 10]

1. Measure Proposal

2. Success: Robust explanation of $\Lambda$.

3. Failure: conflict with local physics and common sense

4. Conclusions
One proposal for regulator:
IR cutoff in bulk from UV cutoff in CFT (see also [Garriga Vilenkin 08 09])

Keep only bulk events whose future lightcones are larger than UV cutoff.
Take continuum limit.
Defines a bulk time variable.
\[ \frac{N_A}{N_B} \] not a good QFT quantity. Dominated by physics at cutoff scale.

If dual theory is an honest QFT, should not try to compute \( \frac{N_A}{N_B} \).

Cutoff proposal not yet well-defined.
Boundary metric only defined up to Weyl transformation \( g_{ab} \rightarrow e^{2\phi(x)}g_{ab} \).

Proposal: Fix ambiguity by demanding \( ^3R = \text{const} \).

Pretty math ensures this is possible: proof of Yamabe conjecture. Generically fixes metric completely.

This procedure defines “new lightcone time.”
Can now characterize attractor behavior.
For small $\Gamma$, volume in each vacuum evolves as

$$\frac{dV_a}{dt} = 3V_a - \Gamma_a V_a + \sum_b \Gamma_{ab} V_b$$

Solutions are exponential. Leading eigenvector determines $\lim_{t \to \infty} \frac{V_a(t)}{V_b(t)}$, independent of initial conditions.
Roughly, volume dominated by longest lived de Sitter vacuum.

Various issues about how well-defined this is, such as

- $\Lambda \leq 0$ regions
- Regions that are not 4-dimensional
When defined, equivalent to counting in one causal patch! For a particular ensemble of patches,

Concretely: initial conditions for causal patch given by attractor behavior.
Take-away message:
One proposal for regulating infinities of eternal inflation $\approx$
Start in longest lived de Sitter vacuum, and restrict to one causal patch.

Encouraging that regulating global description using UV/IR is equivalent
to causal patch description.
1 Measure Proposal

2 **Success:** Robust explanation of $\Lambda$.

3 **Failure:** conflict with local physics and common sense

4 Conclusions
Some success in predictions from theory for
- Cosmological Constant
- Spatial Curvature
- Bubble Collisions
- Dark Matter Abundance

I will describe a robust explanation of $\Lambda$.

Improvements relative to Weinberg prediction:
Infinities regulated in a well-motivated way, better agreement with observation, and more robust.
(Also 25 years later and not a prediction)
Question: Given that we live 13.7 Gyr after big bang (tunneling event), what $\Lambda$ should we expect to observe?

Putting in $t_{\text{obs}}$, but allowing all other physics to vary. Assuming observers in a variety of vacua. Count how many in each vacuum. Precise definition of observer needed, but this analysis is insensitive to definition.

Really addressing coincidence problem: why do we live at the special time when $\Omega_{\text{matter}} \approx \Omega_{\Lambda}$?

Why is $\Lambda \approx t_{\text{obs}}^{-2}$ when natural value is $\Lambda \gg t_{\text{obs}}^{-2}$?

Not addressing why $t_{\text{obs}} \sim 10^{60}/P$ here.
Predict by counting

\[ \frac{dp}{d\Lambda} = \frac{dN_{\text{obs}}}{d\Lambda} = \frac{dN_{\text{bub}}}{d\Lambda} M_{\text{CP}}(\Lambda, t_{\text{obs}}) \alpha(\Lambda, t_{\text{obs}}) \]  

(1)

\(M_{\text{CP}}\) is mass inside causal patch.
\(\alpha\) is number of observers per unit mass.
\[ \frac{dp}{d\Lambda} = \frac{dN_{\text{bub}}}{d\Lambda} M_{\text{CP}}(\Lambda, t_{\text{obs}}) \alpha(\Lambda, t_{\text{obs}}) \quad (2) \]

\[ \frac{dN_{\text{bub}}}{d\Lambda} = 1 \text{ in regime of interest.} \]

Nucleation rates insensitive to late time \( \Lambda \).

Distribution of vacua uniform near \( \Lambda = 0 \) for \( \rho_\Lambda \ll M_{\text{SUSY}}^4 \)

Stupid model: Pretend \( \alpha \) independent of \( \Lambda \).

Overcounts observers in naively natural regime \( \Lambda \gg t_{\text{obs}}^{-2} \).

\[ \frac{dp}{d\Lambda} \propto M_{\text{CP}}(t_{\text{obs}}, \Lambda) \]

Can compute using FRW equation.
\[
\frac{dp}{d\Lambda} \propto M_{\text{CP}} = \begin{cases} 
\Lambda^{-1/2}, & \Lambda \ll t_{\text{obs}}^{-2} \\
\Lambda^{-1/2} \exp\left(-3t_{\text{obs}}\sqrt{\Lambda}\right), & \Lambda \gg t_{\text{obs}}^{-2}
\end{cases}
\] (3)

Geometric explanation for why we don’t observe large \(\Lambda\):
Exponentially small amount of mass inside causal patch, because mass
diluted by exponential expansion.

\(\Lambda \ll t_{\text{obs}}^{-2}\) requires extra tuning.

Explains \(\Lambda \approx t_{\text{obs}}^{-2}\).

No detailed anthropic assumption, good agreement with observation.
Measure Proposal

Success: Robust explanation of $\Lambda$.

Failure: conflict with local physics and common sense

Conclusions
Failure: Disagreement with local physics.

“Sleeper Paradox” [Guth Vanchurin 11] [Bousso BF Leichenauer Rosenhaus 10]: Flip a fair coin. Before you see the outcome of the coin flip, an experimenter puts you to sleep. If the coin is heads, you sleep for 1 minute; if tails, you sleep for 5 billion years.

You wake up with no memory of how long you slept, and are asked to bet whether you slept for a short time or a long time.

Local physics, common sense $\rightarrow p_S = p_L$

Eternal inflation:

$$\frac{p_S}{p_L} = \frac{N_S}{N_L} > 1$$
Either

- Order one corrections to local physics on time scale $H^{-1}$, or
- All geometric cutoffs are wrong.

[Olum 2012]

Vote: Are these order one corrections acceptable?
How to make real world predictions is an important question. One proposal is to make probabilistic predictions from eternal inflation. Requires solving the measure problem.

One measure \( \approx \) start in the longest lived de Sitter vacuum and restrict to one causal patch.

Simple geometric reason we do not observe \( \Lambda \gg t_{\text{obs}}^{-2} \): matter inflated out of causal patch.

Same geometry gives order one corrections to local physics on time scale \( H^{-1} \).
Many more tests of measures as we learn more about landscape. Measures in agreement with observation only if:

- De Sitter vacua decay faster than a nontrivial bound.
- Tunneling rates to vacua with $\Lambda \approx 0$ have certain properties.
- Tuning required for $N_e$ efoldings $\sim N_e^{-p}$

More generally,

- Holographic description of eternal inflation
- Need for more predictions. Some understanding of spatial curvature, dark matter abundance. Particle physics predictions?

Better understanding of complementarity to resolve conflict with local physics.
Q: Does it make sense to make a probabilistic prediction for a quantity like $\Lambda$ we can only measure once?

A: Maybe not, but we do this all the time in physics. Knowing inflaton potential allows only probabilistic prediction for CMB sky (cosmic variance). Only get one CMB sky.
Is there some $\ell$ where we should stop making predictions for CMB?

Technically, the theory of CMB fluctuations is under much better control, and makes more predictions.
Philosphically, the situation is the same.
Have not explained why $t_{\text{obs}} \sim 10^{60}/P$.

Much more speculatively, can allow $t_{\text{obs}}$ to vary as well.

Find power law divergence as $t_{\text{obs}} \to \infty$.

Cut off by discretuum limit.

\[ t_{\text{obs}} = t_{\Lambda}^{\text{max}} = \Lambda_{\text{min}}^{-1/2} \quad (4) \]

\[ \Lambda \approx \Lambda_{\text{min}} \sim \frac{M_P^2}{N_*} \quad (5) \]

where $N_*$ is the number of anthropically acceptable vacua that are populated reasonably well by eternal inflation.
Physical interpretation of strange probabilities:
When observers pass out of causal diamond, time ends for them.
Danger of hitting end of time,

\[ \frac{dp}{d\tau} \sim H \]

“Explains” why \( p_S \neq p_L \). But at cost of introducing crazy new local physics.

Some recent suggestions that horizons are more dangerous than we thought, but my money is still on option 2: existing cutoffs are too crude.