RG and Unitarity in Spacetime-dependent QFT

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I. [1207.xxxx] New Perturbative Critical Phenomena

[1203.1680 PRD] Effect on Unitarity bounds

II. [1208.xxxx] $\ell d_\ell \, \sigma_{\text{single-trace}} = 0$ for (A)dS_D / dS_{D-1} duality

I. Consider

$$S = S_0 + \int d^d x \, \underbrace{\lambda(x\mu)^\kappa}_{} \mu^{d-\Delta} \mathcal{O}_\Delta$$

affects scaling under $x \to x/b$, & relevance or irrelevance at large x

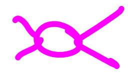
a) Novel fixed points generalizing Wilson-Fisher, but perturbative in physical (integer) dimensionalities (small N!)

b) Shifted unitarity bounds explain IR content of x-dependent QFTs (large N, including FRW duals)

a) Consider $(d = 4-\varepsilon)$

$$S = \int d^d x \left\{ \frac{1}{2}(\partial\phi)^2 - \frac{\lambda x^{\kappa} \mu^{\kappa+\varepsilon}}{4!}\phi^4 \right\}$$

$\kappa = 0$ Wilson/Fisher "Critical Exponents in 3.99 Dimensions"

bare $\hat{\lambda}_0 = \mu^{\varepsilon}\lambda + \frac{3\lambda^2}{16\pi^2}\frac{1}{\varepsilon} + \dots$ β_{4d}  $+ \dots$

$$\Rightarrow \beta_\lambda = -\varepsilon\lambda + \frac{3\lambda^2}{16\pi^2} + \dots$$

- small $\varepsilon \Rightarrow \lambda_* = \frac{16\pi^2}{3}\varepsilon + O(\varepsilon^2)$

  fictional fractional dimension

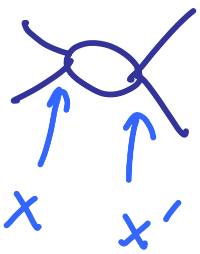
- $\varepsilon = 1$ ε expansion + resummation

$\Rightarrow$ critical exponents for spin systems: QFT $\Leftrightarrow$ data

$$S = \int d^d x \left\{ \frac{1}{2}(\partial\phi)^2 - \frac{\lambda x^\kappa \mu^{\kappa+\varepsilon}}{4!}\phi^4 + \dots \right\}$$

Critical Exponents in 4.0 Dimensions

$\kappa \neq 0$:



$$\sim \lambda^2 \mu^{2(\kappa+\varepsilon)} \int \frac{d^d x_+ \, d^d x_- \, x^\kappa x'^\kappa}{(x_-^2)^{d-2}}$$

$$= \int d^d x_+ \underbrace{\lambda^2 \mu^{2(\kappa+\varepsilon)} x_+^{2\kappa}}_{\lambda(x)^2} \underbrace{\int \frac{d^d x_-}{(x_-^2)^{d-2}}}_{\text{usual log}} + \mathcal{O}\left(\frac{\kappa x_-^2}{x_+^2}\right)$$

Bare coupling

$$\breve{\lambda}_0 = \mu^\varepsilon \left[\lambda \mu^\kappa x^\kappa + \frac{3\lambda^2 \mu^{2\kappa} x^{2\kappa}}{16\pi^2} \frac{1}{\varepsilon} + \dots \right]$$

Bare coupling

$$\tilde{\lambda}_0 = \mu^{\varepsilon}\left[\lambda_\mu{}^{k} x^{k} + \frac{3\lambda^2 \mu^{2k} x^{2k}}{16\pi^2}\frac{1}{\varepsilon} + \dots\right]$$

$$\boxed{e^{-1/k} \ll x_\mu \ll e^{1/k}}$$

$$\to \quad \tilde{\lambda}_0 \simeq \tilde{\mu}^{\varepsilon+k}\left(\lambda + \frac{3\lambda^2}{16\pi^2}\frac{1}{\varepsilon} + \dots\right)$$

$$\beta = \mu\frac{\partial\lambda}{\partial\mu} = -\underbrace{(\varepsilon+k)}_{\to\, 0\ (d=4)}\lambda + \frac{3\lambda^2}{16\pi^2} + \dots$$

$$\Rightarrow \quad \lambda_* \simeq \frac{16\pi^2}{3}k \quad \text{in } 4d$$

* Fixed point under $X_\pm \to X_\pm / b$

cf Osborn, Lawrence/Sever...

This gave a fixed point to very good approximation over exponentially wide range of scales.

- $\exists$ exact fixed point with further tuning

$$\lambda = \lambda_0 + \lambda_1 \, \kappa \log \mu x + \lambda_2 (\kappa \log \mu x)^2 + \cdots$$

$$\rightarrow \quad \beta_{\lambda_0} = -\kappa \lambda_1 + \frac{3\lambda_0^2}{16\pi^2} + \cdots$$

$$\beta_{\lambda_1} = -\kappa \lambda_2 + \frac{6\lambda_0\lambda_1}{16\pi^2} + \cdots$$

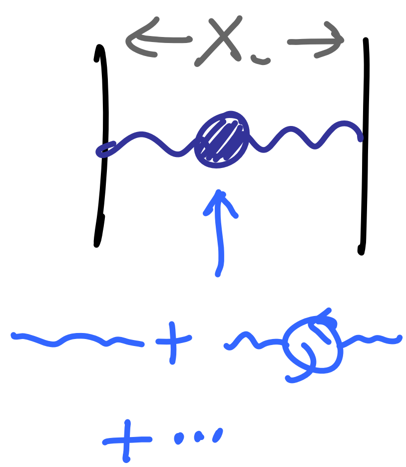
(iterates: $\lambda_{m*} \stackrel{\sim}{=} c \, \lambda_0 \lambda_{m-1}$)

- Similar mechanism for any theory with marginally (ir)relevant coupling

- KQED* ($m_{e^-} \to 0$)

$$\beta = -\kappa e + \frac{e^3}{12\pi^2} + \cdots$$

Force between charges:



$$V(x_-) \simeq \frac{\alpha}{x_-}\left[1 + 2\kappa \log(x_+\mu) - \frac{\alpha}{3\pi}\log(x_-^2\mu^2)\right]$$

$$V \to Vb \qquad x_\pm \to x_\pm / b$$

* supported by viewers like you.

- KQCD $(K \to -K)$
- $d=3$ $\lambda_6 (\vec{\phi}^2)^3$ $O(N)$ theory
  (tricritical He4 - He3 etc)

$$\beta = -K\lambda_6 + \frac{22+3N}{240\pi^2}\lambda_6^2 + \cdots$$

- $d=2$ σ - models

$$\beta_{G_{ij}} = -K G_{ij} - R_{ij}$$

fixed point $R_{ij} = -K G_{ij}$

(Einstein space)

• Strongly coupled examples

$$S_{(s)CFT} + \int \lambda(x)\, \mathcal{O}_{\text{marginal}}$$

$$\rightarrow \beta \simeq -K\lambda + C\lambda^2$$

$$\langle \mathcal{O}\mathcal{O}\mathcal{O} \rangle_{CFT}$$

General mechanism; examples include small - N_f, N_c theories w/o SUSY, perturbatively controlled

- could be tested
- killer app?

b) Unitarity Bounds

Constrain IR physics

$$\Delta S = \int \lambda(x_\mu)^K \mu^{d-\Delta} \mathcal{O}_\Delta$$

makes $\mathcal{O}_\Delta$ relevant

for $K > \Delta - d$

This changes IR content of the theory, and (as we will see) unitarity bounds adjust accordingly.

Consider

$$\int \Delta \mathcal{L} = \int dt\, d^{d-1}\vec{x}\; g(t,\vec{x})\, \mathcal{O}$$

$$g = g_0 t^K \quad \text{or} \quad g_0 (t^2 - \vec{x}^2)^{K/2}$$

as $t \to \infty$

- K can change whether $\Delta\mathcal{L}$ dominates at late times (IR)
- If $\Delta\mathcal{L}$ marginal in IR under $x^\mu \to \lambda x^\mu$, then

$$[\mathcal{O}] = d + K$$

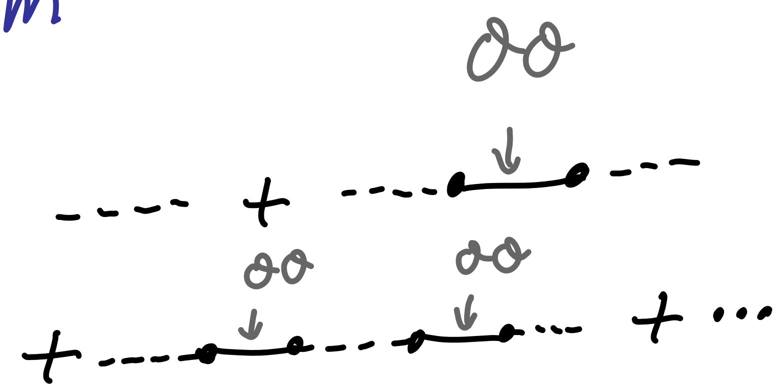
$\to$ Expect K can shift relevance condition & unitarity bounds

We can analyze this explicitly in large-N double trace flows.

$$\Delta S = \int \tfrac{1}{2}\phi m^2 \phi + g(t,\vec{x})\,\mathcal{O}\phi$$

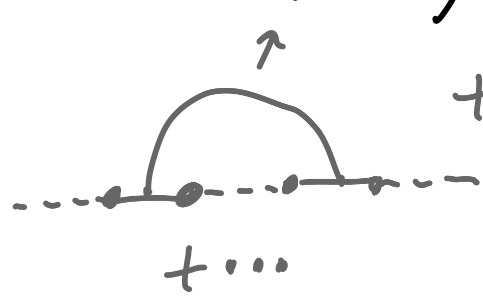
$$\simeq \int \frac{g^2}{2m^2}\mathcal{O}\mathcal{O}$$

$\langle \phi\, \phi \rangle =$



Effectively Gaussian

$+\ \mathcal{O}\left(\frac{1}{N}\right)$



$+$ non-adiabatic effects

Static Limit:

cf Andrade et al

$$\Delta_\pm = \frac{d}{2} \pm \nu$$

$$\langle \mathcal{O}_\pm(p)\, \mathcal{O}_\pm(-p) \rangle = -i\, C_{\pm\nu} (p^2 - i\varepsilon)^{\pm\nu}$$

$$S_{CFT}^{(+)} + \int \frac{g^2}{4m^2} \underbrace{\mathcal{O}_+ \mathcal{O}_+}_{\substack{\Delta = d+2\nu \\ \text{irrelevant}}}$$

$$\langle \phi(p)\, \phi(-q) \rangle = \frac{-i\, \delta(p-q)}{m^2 - g^2 C_\nu (p^2)^\nu}$$

$$\boxed{\frac{2^{-2\nu} \pi^{\frac{d}{2}}\, \Gamma(-\nu)}{\Gamma(\frac{d}{2}+\nu)}}$$

$\rightarrow \langle \mathcal{O}_- \mathcal{O}_- \rangle$

as $p^2 \rightarrow \infty$

UV $\mathcal{O}_-$

nonunitary for $\nu > 1$

$\Lambda_g^n \quad \dfrac{1}{g^{\frac{1}{\nu-1}}}$

ok as cut off QFT

IR $\mathcal{O}_+$ unitary

$\boxed{\text{t-dependent case}}$

- $\int \lambda_0 t^{2K} \mathcal{O}_+^2$ is <u>relevant</u>

(dominates 2 pt ftns at large x)

when $[\lambda_0] = 2(K-\nu) > 0$

- Unitarity maintained, including $\nu > 1$

- Can UV complete, e.g. SUSY models

$$\Delta S = \int \tfrac{1}{2}\phi\, m^2 \phi + \underbrace{g(t,\vec{x})\,\phi}_{\equiv\ \tilde{\phi}}\, \mathcal{O}_+$$

$$\langle \tilde{\phi}(p)\, \tilde{\phi}(-q) \rangle \xrightarrow[\mathrm{IR}]{} i\,\delta(p-q)\, \frac{(p^2 - i\varepsilon)^{-\nu}}{C_\nu}$$

* Not norm of a state

$$\Rightarrow \langle \phi(x)\, \phi(x') \rangle = \frac{-1}{C_\nu C_{-\nu}\, g(x)\, g(x')\, \left[(x-x')^2\right]^{\Delta_-}}$$

*

Despite the Δ_- here, forward scattering* 

is unitary

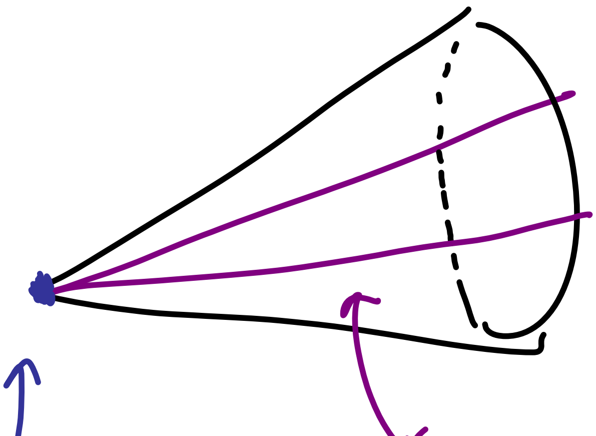
$$\mathrm{Im}\, A(\chi \to \chi) \propto -\sin \pi\nu / C_\nu > 0$$

(Intriligator/Grinstein: $\mathrm{Im}\, A_{\mathcal{O}} \propto C_{\mathcal{O}} \left(\Delta - \left(\frac{d-2}{2}\right)\right)$ in CFTs)

Our original motivation: DHMST'11

duals of FRW backgrounds:

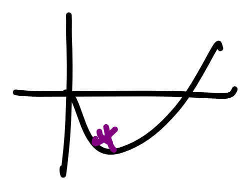
Seiberg Witten, Sen,
Banks Douglas Seiberg
Argyres Douglas Shenker,
Aharony Kachru ES
Aharony Fayyazuddin Maldacena
Polchinski ES



D3

(p,q) 7-branes

$n_7 < 36 \longrightarrow$ Static AdS/CFT Solution

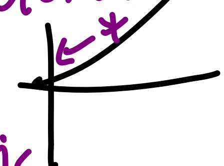


$\uparrow$

flavor content

$\downarrow$

$n_7 > 36 \longrightarrow$ time-dependent solution

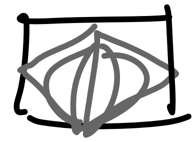


still with warped metric

36 = <u>static</u> unitarity bound

II. Holographic RG for $(A)dS_D/dS_{D-1}$

dS_D/dS_{D-1} Correspondence:

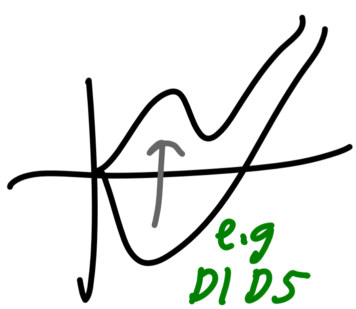


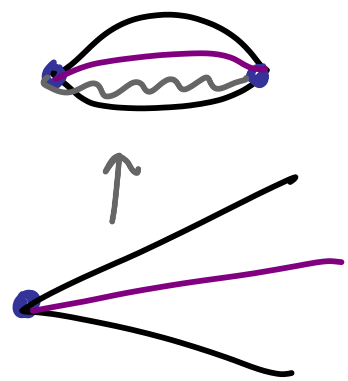
Macro $$ds^2_{(A)dS_D} = \sin(h)\left(\frac{w}{L}\right)^2 ds^2_{dS_{D-1}} + dw^2$$

2 highly redshifted regions $\Rightarrow$ EFTs (e.g. Liouville)

UV cutoff + GR_{D-1}

Micro





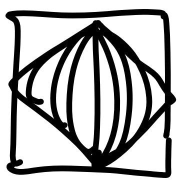
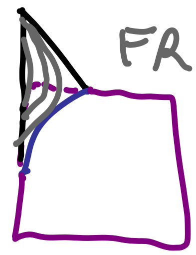
Same structure : 2 large-$N_1 N_5$ matter sectors

$\rightarrow$ parametric estimate of S_{dS}

(= # of degrees of freedom needed to reconstruct static patch)

Comparative holography:

Recall in AdS/CFT that p-brane construction lands on Poincaré slicing. In dS + FRW, above brane constructions → slicings

dS/dS  &  FRW/t-dependent QFT

- inside a causal region
- a spatial direction (↔ scale) emerges.

⇒ • # of degrees of freedom real, >0 and unitarity more transparent
- Symmetries (such as they are) less manifest

As in AdS → Global, this may connect to other slicings (dS/CFT, FRW/CFT)

also have dynamical gravity

Anninos/Hartman/Strominger
Harlow/Shenker/Stanford/Susskind

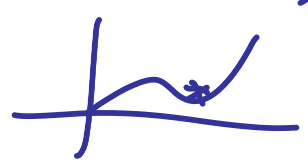
② UV structure of dS_D duals

Maximal symmetry & RG

$$ds_D^2 = \sin(h)^2\frac{w}{L}\, ds^2_{dS_{D-1}} + dw^2$$

(1) In AdS_D, Moduli fixing ($\partial_{\Phi_I} V=0$) is dual to $\beta_{\{\lambda\}}=0$.

(2) In dS_D, we also (meta-)stabilize the moduli ($\partial_{\Phi_I} V=0$)



$M^2 \geq 0$

→ How are (1) & (2) reflected in holographic RG? What is the (D-1) dual of (2)?

Wilsonian holographic RG

... Heemskerk
Polchinski

$$\begin{cases} ds^2 = L_D^2 \dfrac{dz^2}{z^2} + a(z,x)^2 ds^2_{dS_{D-1}} \\ \text{scalar } \phi \text{ with potential } V(\phi) \end{cases}$$

$a(z,x) \leftrightarrow T$

$\phi \leftrightarrow \mathcal{O}$

integrate out

$z=\ell$ $\quad$ $z=\varepsilon$

$\to$ Wilson action $S(\ell) = \int \tilde{a}^d_{(\ell)}\, \sigma_{nm}(\ell)\, T^n \mathcal{O}^m$

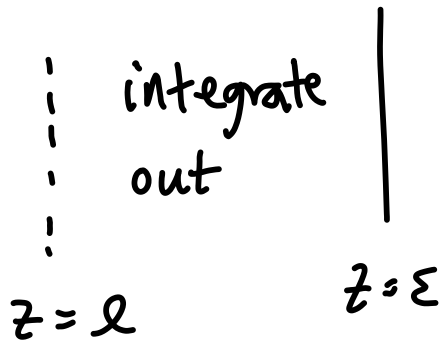
- For $V'(\phi_*) = 0$, find solution $\partial_\ell \sigma_{01}^{(\vec{L}=0)} = 0$ regardless of sign of $V_* = V(\phi_*)$

$\to$ single-trace coupling ℓ-independent for both dS_D & AdS_D

Metastability & RG

· KKLT, MSS, ...

· AdS/CFT uplifts

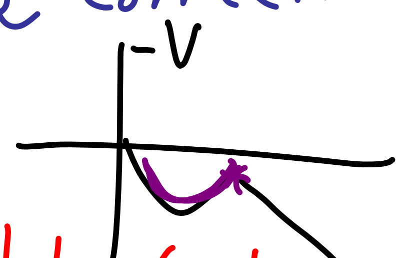


radial Hamiltonian

$$\psi_{UV}[\hat\phi, \hat a; \ell] = \langle \hat\phi\, \hat a \,|\, e^{-H \log \ell} \,|\, \phi_0\, a_0 \rangle$$

(related to Wilson action by integral transform)

Meta-stability adds bounce solution, $\Rightarrow$ non-perturbative correction to Wilson action



$\rightarrow$ dS alone incomplete (not unitary), decays $\rightarrow$ FRW.

Summary

I. Couplings $\sim \int \lambda(x_\mu)^\kappa \mathcal{O}$

affect IR physics, leading to novel perturbative fixed points and shifted unitarity bounds.

II. Maximal symmetry $(V'=0)$

$\Rightarrow \quad \mu \partial_\mu \sigma_{\text{single-trace}} = 0$

in dS/dS holography.