Interfaces in Landau Ginzburg theories

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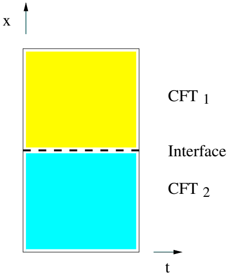
Outline

- Interfaces and their fusion
- Interfaces and bulk perturbations
- D-branes and interfaces in Landau-Ginzburg models
- "Flow interfaces" for N=2 minimal models
- Interfaces realizing mondromy transformations for Calabi-Yau compactifications

Based on work with Daniel Roggenkamp 0707.0922, 0712.0188 and with Hans Jockers and Daniel Roggenkamp 0806.4734

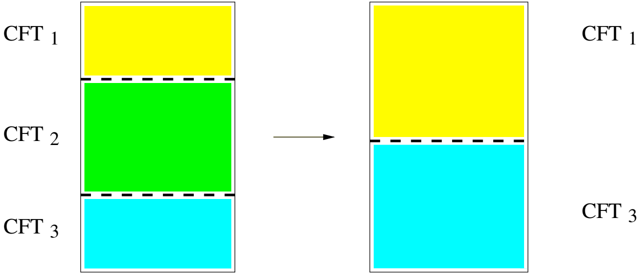
Interfaces for conformal field theories



- Two conformal field theories are joined along a common interface, which is required to preserve conformal invariance

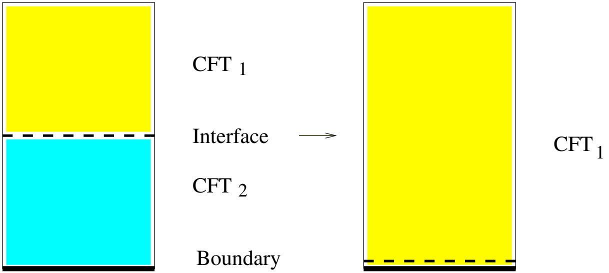
  Bachas-de Boer-Dijkgraaf-Ooguri, Petkova-Zuber, Fröhlich-Fuchs-Runkel-Schweigert
- The interface can carry additional degrees of freedom which are not inherited from the bulk.
- Special cases
  - Totally reflecting: Interface is a boundary for both theories.
  - Totally transmitting: Topological interface, e.g. trivial.

Fusion of interfaces



- Two interfaces merge to form a new interface. In general:singular process.
- In the limit, obtain a new interface between theory CFT_1 and CFT_3.

Action on boundary conditions



- Special case: Interfaces can fuse with boundaries to form new boundaries.
- Interfaces act naturally on D-branes.

Topological interfaces

- Topological interfaces are interfaces for which the stress energy tensor for the left and right movers is continuous on the full complex plane (or general world sheet)

$$T^{(1)} = T^{(2)}, \ \bar{T}^{(1)} = \bar{T}^{(2)} \text{ on the interface} \ .$$

  which means that the correlation functions are still covariant with respect to any conformal transformation of the world sheet.

- The interface can hence be deformed or moved across the world sheet in arbitrary ways – as long as it does not hit a field insertion; that is the reason they are called topological.
- Since topological interfaces can be deformed and moved in arbitrary ways, they can be fused smoothly.

Conformal interfaces

- More generally, we can just require that $T - \bar{T}$ is continuous across the interface.

$$T^{(1)} - \bar{T}^{(1)} = T^{(2)} - \bar{T}^{(2)}$$

Such interfaces are called conformal interfaces.

- A special class are the totally reflecting interfaces, which are boundaries for the two theories. In this case the rhs and lhs of the above equation vanish separately at the boundary.
- The fusion of conformal interfaces is in general a singular process that needs regularization. This has been worked out for the free boson. Bachas-Brunner.
- In this talk, we will circumvent these problems by working in topologically twisted theories.

The folding trick

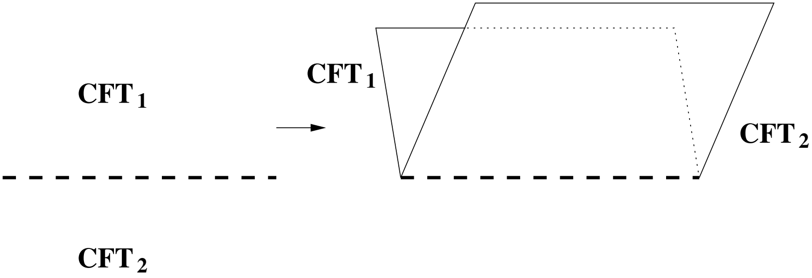
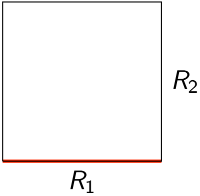


Figure: Folding trick.

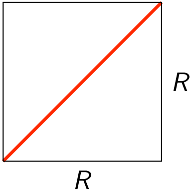
- Instead of a theory on the full plane with an interface along the real line and theories CFT_1 and CFT_2 on the upper and lower half plane, one can consider the theory $\mathrm{CFT}_1 \otimes \overline{\mathrm{CFT}}_2$ on the upper half plane. Here, $\overline{\mathrm{CFT}}_2$ is obtained from CFT_2 by exchanging left and rightmovers.

Interfaces for the free boson compactified on a circle

- Using the folding trick, we can picture the interfaces as boundary conditions (D-branes) for two free bosons ϕ_1, ϕ_2 compactified on $S^1 \times S^1$.
  - Totally reflecting interface

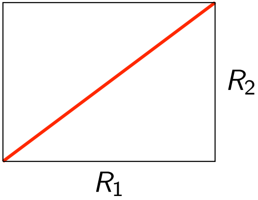


  - Identity interface

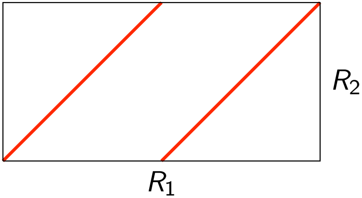


More interface pictures for the free boson

- Basic radius changing interface



- Multiple windings



Why interfaces and fusion?

- In string theory, we know of many operations on D-branes
  - T-duality, mirror symmetry
  - Calabi-Yau compactifications: Monodromies around singular points in moduli space.
  - Bulk perturbations: The D-brane has to adjust to the new background.
  - Open string tachyon condensation. Bachas-Gaberdiel.
- All of these seem to be described by interfaces. Interfaces are useful tools to study some of these operations.

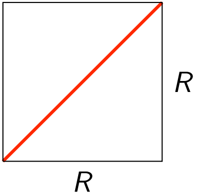
Interfaces and bulk induced boundary RG flows

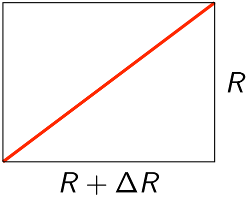
with Daniel Roggenkamp

- Consider CFT on a disk. Turn on a relevant (or marginal) bulk perturbation. The perturbation will induce a flow in both the bulk and boundary sector; UV boundary conditions will flow to boundary conditions of the IR theory. Given a boundary condition for the UV theory, which boundary condition in the IR will it flow to?
- String theory language: Behavior of branes under closed string tachyon condensation.
- "Standard" procedures: Coupled bulk-boundary RG flow.
- Interfaces can provide a new method to handle the regularization and renormalization. Here, bulk and boundary flow become decoupled.

Flow interfaces – the folded theory

- How does one construct an interface connecting a perturbed and an unperturbed theory?
- Start with the identity defect/diagonal brane and perturb





Flow interfaces – unfolded

- Regularization of bulk-boundary flows using interfaces: Two step procedure
  - Restrict the perturbation to a subset of the Disk. Obtain an interface separating IR and UV of the theory.
  - Bring the interface to the boundary. This is a singular procedure, the interface will not be topological.

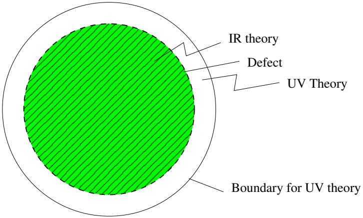


Figure: Perturbation restricted to a domain U (shaded). UV and IR theory are separated by an interface line.

Discussion of the method

- The interface connects directly UV and IR of the theory. The interface approach is hence non perturbative (in the bulk couplings).
- The fusion process is complicated. However, one can consider $N = (2, 2)$ supersymmetric models, which can be topologically twisted. On the level of the topological theory, the interfaces compatible with the twist are topological, and one can work out the fusion.

D-branes in $N = (2, 2)$ theories

- $N = 2$ theory has currents $T, J, G^{\pm}$.
- A-type boundary conditions (boundary at real line)

$$T(z) = \overline{T}(\bar{z}), \quad J(z) = -\overline{J}(\bar{z}), \quad G^{\pm}(z) = \pm\bar{G}^{\mp}(\bar{z}), \quad z = \bar{z}$$

- In a Calabi-Yau compactification, these D-branes correspond to special Lagrangian submanifolds.
- B-type boundary conditions (boundary at real line)

$$T(z) = \overline{T}(\bar{z}), \quad J(z) = \overline{J}(\bar{z}), \quad G^{\pm}(z) = \pm\bar{G}^{\pm}(\bar{z}), \quad z = \bar{z}$$

- Geometrically, holomorphic submanifolds.

Interfaces in $N=(2,2)$ theories

- A-type interface

$$
\begin{aligned}
T^{(1)} - \overline{T}^{(1)} &= T^{(2)} - \overline{T}^{(2)}\,, \\
J^{(1)} + \overline{J}^{(1)} &= J^{(2)} + \overline{J}^{(2)}\,, \\
G^{\pm(1)} + \overline{G}^{\mp(1)} &= G^{\pm(2)} + \overline{G}^{\mp(2)}
\end{aligned}
$$

- B-type interface

$$
\begin{aligned}
T^{(1)} - \overline{T}^{(1)} &= T^{(2)} - \overline{T}^{(2)}\,, \\
J^{(1)} - \overline{J}^{(1)} &= J^{(2)} - \overline{J}^{(2)}\,, \\
G^{\pm(1)} + \overline{G}^{\pm(1)} &= G^{\pm(2)} + \overline{G}^{\pm(2)}
\end{aligned}
$$

- Folding trick: Description in terms of D-branes on $M \times M'$, where M, M' are compactification manifolds.

Supersymmetry preserving perturbations

- Two types of perturbations that preserve SUSY in the bulk
  - (c,c) perturbations $\Delta S = \int d^2x d\theta^+ d\theta^- \Phi|_{\bar\theta^\pm=0}$
  - (a,c) perturbations $\Delta S = \int d^2x d\theta^+ d\bar\theta^- \Psi|_{\bar\theta^+=\theta^-=0}$
- In a Calabi-Yau compactification, these correspond to complex structure and Kähler deformations.
- In theories with boundaries, supersymmetry can be preserved if the perturbation is (c,c) [(a,c)] and the boundary is A-type [B-type].Hori-Iqbal-Vafa
- Expect that (c,c) [(a,c)] perturbations are described by A [B] type defects, and the behavior of D-branes under such perturbations can be described by fusion.
- This works for A-branes in $N=2$ minimal models, using defects on the mirror B side, and fusing them with the mirror B-branes. (relevant perturbations.)
- Monodromies of LG B-branes in the Kähler moduli space provide a second example, where the perturbation is marginal.

Landau Ginzburg models with boundaries

- Landau Ginzburg action
$$S = \int d\theta^+ d\theta^- W(X),$$
where X is a chiral superfield, $\bar{D}_\pm X = 0$
- This action is invariant under $N = (2,2)$ SUSY for worldsheets without boundary. If there is a boundary, the B-supersymmetry variation will produce a boundary term $\sim \int_{\partial\Sigma} dtd\theta W$.
- Introduce a boundary F-term
$$\Delta S = \int_{\partial\Sigma} J(X)\pi \ ,$$
where π is a boundary fermion. It is not chiral, but fulfills $\bar{D}\pi = E(X)$. The variation of the boundary term thus cancels the unwanted term resulting from the variation of the bulk action if
$$J(X)E(X) = W(X)$$

Matrix factorizations of the superpotential

- Boundary BRST operator

$$Q = \begin{pmatrix} 0 & E \\ J & 0 \end{pmatrix} = \pi J + \bar{\pi} E \qquad Q^2 = W \ .$$

The open string Hilbert space is the cohomology of this BRST operator. Bosons are given by block-diagonal matrices, fermions by off-diagonal matrices.

- One often represents matrix factorizations in the following way:

$$P : \quad P_1 = \mathbb{C}[X_i]^N \underset{p_0}{\overset{p_1}{\rightleftarrows}} \mathbb{C}[X_i]^N = P_0 \, ,$$

$$p_1 p_0 = W(X_i)\mathrm{id}_{P_0} \, , \quad p_0 p_1 = W(X_i)\mathrm{id}_{P_1}$$

Equivalence of matrix factorizations

- The factorization $W = W\ 1$ is trivial in the sense that its spectrum with any other factorization is empty. We can hence add such branes to any other stack of branes without changing the physics.

$$Q_{\rm bd} \sim Q_{\rm bd} \oplus Q_{\rm triv}$$

- Two boundary conditions specified by BRST charges $Q_{\rm bd}$ and $Q'_{\rm bd}$ are equivalent if

$$Q'_{\rm bd} = U Q_{\rm bd} V\,, \qquad UV = {\rm id}' + \{Q'_{\rm bd}, O'\}\,, \quad VU = {\rm id} + \{Q_{\rm bd}, O\}$$

- The BRST cohomology does not change under these similarity transformations.
- U and V can be regarded as open string operators propagating from one brane to another in opposite directions. The condition above can be read as the existence of an identity operator in the open string spectrum.

Mathematical origins

- All these ideas had been developped mathematics. Landau-Ginzburg theories are closely linked to singularity theory, and matrix factorizations have played a prominent role there. The idea to describe D-branes in LG models via matrix factorizations is due to Kontsevich. Physics: Kapustin-Li, Brunner-Herbst-Lerche-Scheuner, Lazaroiu

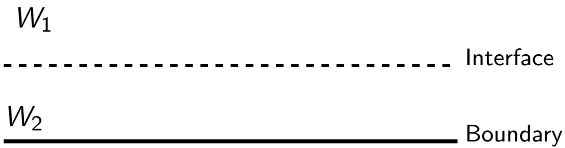
- Action

$$S_F = \int_{UHP} d^2x d\theta^- d\theta^+ W_1(X_i) + \int_{LHP} d^2x d\theta^- d\theta^+ W_2(Y_i) + cc$$

- In the case of interfaces, the supersymmetry variation gives a Warner term for both the theories on the upper and lower half plane, with a relative sign because of the different relative orientations. See also Khovanov-Rozansky
- Hence, interfaces in Landau-Ginzburg models are described by matrix factorizations of the difference of the superpotentials $W = W_1 - W_2$.
- This is in perfect agreement with the folding trick, which tells us to look for boundary conditions in the tensor product of theory 1 and theory 2, where left and right movers have been exchanged for theory 2.

$$\int d\theta^+ d\theta^- W_2(Y) \to -\int d\theta^+ d\theta^- W_2(Y)$$

Action on boundary conditions



- Interface and boundary condition are specified by BRST operators $Q_{\rm def}$ and $Q_{\rm bd}$ satisfying

$$Q_{\rm def}^2 = (W_1 - W_2), \quad Q_{\rm bd}^2 = W_2$$

- Taking the limit where the interface coincides with the boundary all fermionic degrees of freedom are moved to the new boundary. The new BRST charge is

$$Q'_{\rm bd} = Q_{\rm def} + Q_{\rm bd}$$

- The new boundary condition is obtained from the old one by a tensor product construction.

The new BRST charge

- The sum of the BRST charges has indeed the right properties to describe a boundary condition for the theory with superpotential W_1.

$$(Q'_{\rm bd})^2 = Q_{\rm def}^2 + Q_{\rm bd}^2 = W_1(X_i) - W_2(Y_i) + W_2(Y_i) = W_1(X_i)$$

- Similarly, two interfaces can be composed by adding the BRST charges.
- The variables Y_i still appear in the factorization of a superpotential $W(X_i)$ that no longer depends on those variables. A priori, the factorization is infinite dimensional over $\mathbf{C}[X]$.

$$\mathbf{C}[X, Y] \equiv \mathbf{C}[X] + Y\mathbf{C}[X] + Y^2\mathbf{C}[X] + \dots$$

- One can show that it is equivalent to one that only depends on the X_i. The equivalence involves stripping off infinitely many trivial brane-anti-brane pairs.

Fusion products of interfaces in LG models

- Summarizing, the fusion product between interfaces in LG models (and the oepration of interfaces on branes) is given by taking a tensor product of Chan-Paton spaces plus integrating out the “squeezed in” fields.
- The operation is not commutative, but associative.

RG flow between N=2 minimal models

- We want to use interfaces to describe RG flow between different minimal models, specified by superpotentials $W = X^{d'}$
- Suitable (relevant) bulk perturbations are given by elements of the (c, c) ring, in LG language by monomials of lower degree.

$$W = X^{d'} + \lambda X^{d}$$

  IR: new homogeneous potential $W = X^{d}$.
- This flow is described by an A-type interface.
- We consider the mirror theory $W = X^{d'}$ mod $\mathbb{Z}_{d'}$, where the perturbing operators are (a, c) fields from the twisted sector and the interface is B-type.
- They are given by matrix factorizations of $W = X^{d} - Y^{d'}$ divided by $\mathbb{Z}_d \times \mathbb{Z}_{d'}$.

Interfaces between $W = X^d$ mod $\mathbb{Z}_d$ and $Y^{d'}$ mod $\mathbb{Z}_{d'}$

- Consider first

$$W = X^d - Y^d = \prod_{\zeta}(X - \zeta Y) \quad \zeta^d = 1$$

By grouping the linear factors into two subsets, one obtains a factorization of W.

- $J = X - Y$: identity, $J = X - \zeta Y$ implements $\mathbb{Z}_d$ symmetry
- Matrix-version of this construction

$$(X^d - Y^{d'})\mathrm{id}_d = \prod_{\zeta}(X\mathrm{id}_d - \zeta\Sigma_n)$$

where

$$(\Sigma_n)_{a,b} = \delta^{(d)}_{a,b+1} Y^{n_a}, \quad \sum n_a = d', \quad \Sigma_n^d = Y^{d'}\mathrm{id}_d$$

- A single linear factor in the above factorization looks like

$$\begin{pmatrix} X & & & -Y^{n_0} \\ -Y^{n_1} & \ddots & & \\ & \ddots & \ddots & \\ & & -Y^{n_{d-1}} & X \end{pmatrix}, \qquad \sum n_a = d'$$

Flow interfaces between minimal models

- In fact, the interfaces constructed above are the "flow interfaces" that describe the RG flow between different minimal models.
- Fusing two of them, one obtains an interface of the same class.
- Any flow interface can be obtained by composing single-step interfaces between $W = X^{d+1}$ and $W = X^d$.
- The fusion of the flow interfaces with boundaries describes the boundary RG flow. Agreement with previous results for $d' = nd$ Gaberdiel-Lawrence
- One can make the idea that these interfaces are flow interfaces more precise by considering a linear-sigma-model type prescription: Here, one would consider a model with $W = P^{d'} X^d$ and construct an identity defect there. Flowing to the models $W = P^{d'}$ and $W = X^d$ yields the above interfaces.

Monodromy defects for Calabi-Yau compactifications

- Special points in the Calabi-Yau moduli space: Conifold, LG-point, large volume point.
- Defects in Landau-Ginzburg models should describe monodromies based at the LG point.
- The monodromy around the Landau-Ginzburg point is simply given by the quantum symmetry. A defect implementing this symmetry transformation can easily be constructed.
- Conifold: At the conifold point, the volume of the whole Calabi-Yau collapses to zero size and D6-anti-D6 pairs can be created for free. They can form boundstates with the probe.
- The conifold defect is therefore roughly

$$D_C = 1 + |D6 >< D6|$$

  Applied to a probe, this gives back the probe and some D6 branes.

The defect changing tachyon for the conifold monodromy

- We want the probe brane to form bound states with the D6 branes, and therefore need to turn on a defect changing tachyon that will then produce the tachyons necessary for bound state formations between the branes.
- The defect changing tachyon should be "universal".

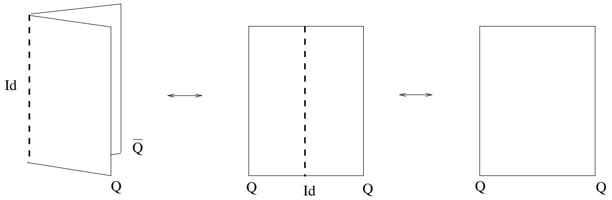


Figure: The spectrum of boundary fields of a boundary condition Q (right) always contains the identity. This yields a universal element in the spectrum of defect-changing fields between the totally reflective defect and the identity defect (left).

Monodromy defects and large volume

- We have constructed the conifold and Gepner monodromy at the LG point.
- The large volume monodromy can be constructed by composing the defects.
- Mathematicians have constructed these monodromies as Fourier-Mukai transformations. For a compactification on a CY M, those are specified by kernels in $D^\flat(M \times M)$.
- Lifting our defects to the linear sigma model, we can move our defects to large volume and find agreement with the kernels of the Fourier Mukai transformations.

Monodromy defects and large volume

- The linear sigma-model construction (Herbst-Hori-Page) allows us to transport branes (defects) along different paths to large volume. In particular, we can transport the identity-defect to large volume on a path that encircles the conifold point. The result is the FM kernel for the conifold monodromy.
- In this way, we have constructed the conifold monodromy on the level of the full topological theory (not just K-theory/RR-charges) without knowledge of the physics of the conifold point (which brane becomes massless), mirror symmetry, Picard Fuchs equations etc.

Conclusions

- Defects/interfaces are useful when studying bulk perturbations. We have seen this for RG flows between N=2 minimal models.
- The same technique could presumably be used for studying closed string tachyon condensation of orbifolds that are not space time supersymmetric. The behavior of D-branes under such deformations has been difficult to analyze using other methods. Martinec-Moore etal.
- Defects describe monodromies in Calabi-Yau moduli spaces. These are special examples of Fourier-Mukai transformations. Presumably, all FMT are defects, and the relation could be made completely precise.