Exercise for Scattering Amplitude (F, T6)
Problem set 8, due to 10 July, 2019

1) Consider the four–gluon MHV amplitude and the following BCJ representation for the three subamplitudes

$$\boldsymbol{A_4(1,2,3,4) = ig_{YM}^2 \left(\frac{n_s}{s} + \frac{n_u}{u}\right) ,}$$

$$\boldsymbol{A_4(1,3,4,2) = ig_{YM}^2 \left(-\frac{n_t}{t} - \frac{n_s}{s}\right) , \ (*)}$$

$$\boldsymbol{A_4(1,4,2,3) = ig_{YM}^2 \left(\frac{n_t}{t} - \frac{n_u}{u}\right) ,}$$

with the three numerators $\boldsymbol{n_s, n_u}$ and $\boldsymbol{n_t}$ fulfilling the numerator equation $\boldsymbol{n_u + n_t = n_s}$. The full amplitude $\mathcal{A}_4$ reads

$$\boldsymbol{\mathcal{A}_4 = \frac{c_s n_s}{s} + \frac{c_u n_u}{u} + \frac{c_t n_t}{t} ,}$$

with kinematic factors fulfilling the Jacobi relation $\boldsymbol{c_u + c_t = c_s}$. The same equations are supposed to hold for an other set of numerators $\boldsymbol{\tilde{n}_s, \tilde{n}_u}$ and $\boldsymbol{\tilde{n}_t}$.

a) We may choose the following numerators:

$$n_s = \frac{\langle 12\rangle^4 [21]}{\langle 23\rangle\langle 34\rangle\langle 41\rangle}, \ n_u = 0, \ n_t = \frac{\langle 12\rangle^4 [21]}{\langle 23\rangle\langle 34\rangle\langle 41\rangle} .$$

As a consequence

$$A_4(1^-, 2^-, 3^+, 4^+) = ig_{YM}^2 \ \frac{n_s}{s} ,$$

i.e. it appears that for this subamplitude there are no contributions from contact terms nor terms accounting for the u–channel. To which amplitude computation does this choice correspond to ? Verify all relations $(*)$ by using the explicit amplitude expressions for the three MHV four–gluon subamplitudes A_4.

b) We may cast $A_4(1^-, 2^-, 3^+, 4^+)$ into the following form:

$$A_4(1^-, 2^-, 3^+, 4^+) = -i \ g_{YM}^2 \ \frac{\langle 12\rangle^2 \ [34]^2}{su} \quad (**) .$$

Prove this relation $(**)$. Determine a set of numerators $\tilde{n}_s$ and $\tilde{n}_u$ (subject to $\tilde{n}_u + \tilde{n}_t = \tilde{n}_s$), which appears naturally after partial fractioning the expression $(**)$. With these numerators verify all relations $(*)$ by using the explicit amplitude expressions for the three MHV four–gluon subamplitudes A_4.

c) Compute the full amplitude $\mathcal{A}_4$ for both sets of numerators n_i and $\tilde{n}_i$ and verify that both expressions are the same.