Exercise for Scattering Amplitude (F, T6)

Problem set 6, due to 26 June, 2019

1) Consider the six–gluon split–helicity NMHV amplitude $A_6(1^-, 2^-, 3^-, 4^+, 5^+, 6^+)$

a) How many diagrams contribute to the recursion relations following from the $[1, 2\rangle$ shifts, i.e.: $\hat{\tilde{\lambda}}_1 = \tilde{\lambda}_1 + z\tilde{\lambda}_2$ and $\hat{\lambda}_2 = \lambda_2 - z\lambda_1$?

b) Compute the contributions of the non–vanishing diagrams and show:

$$A_6(1^-, 2^-, 3^-, 4^+, 5^+, 6^+) = g_{YM}^4 \left\{ \frac{\langle 3|p_1 + p_2|6]^3}{[16][21]\langle 34\rangle\langle 45\rangle\ \langle 5|p_1 + p_6|2]}\ \frac{1}{(p_6 + p_1 + p_2)^2} \right\}$$
$$+ \frac{\langle 1|p_5 + p_6|4]^3}{[23][34]\langle 56\rangle\langle 61\rangle\langle 5|p_1 + p_6|2]}\ \frac{1}{(p_5 + p_6 + p_1)^2} \Big\}\ , \quad (*)$$

with $\langle a|p_b + p_c|d] = \langle ab\rangle[bd] + \langle ac\rangle[cd]$.

c) What is the bahaviour of $(*)$ for large z ?

d) Show, that the expression $(*)$ equals to the result with $[3, 4\rangle$ shifts from the lecture:

$$A_6(1^-, 2^-, 3^-, 4^+, 5^+, 6^+) = g_{YM}^4 \left\{ \frac{\langle 1|p_2 + p_3|4]^3}{[23][34]\langle 56\rangle\langle 61\rangle\langle 5|p_3 + p_4|2]}\ \frac{1}{(p_5 + p_6 + p_1)^2} \right.$$
$$\left. + \frac{\langle 3|p_4 + p_5|6]^3}{[12][61]\langle 34\rangle\langle 45\rangle\ \langle 5|p_3 + p_4|2]}\ \frac{1}{(p_6 + p_1 + p_2)^2} \right\}\ .$$

e) How many diagrams contribute to the BCFW recursion relations following from the $[2, 1\rangle$ shifts ?