# Exercise for Scattering Amplitude (F, T6)
## Problem set 4, due to 5 June, 2019

**1) For the helicity choice $1^-, 2^-, 3^+$ and $4^+$ we have:**

$$A_4(1^-,2^-,3^+,4^+) = g_{YM}^2 \, \frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle} \, .$$

a) After using cyclic invariance there are six four–gluon MHV amplitudes. What are the the other five amplitudes starting with $1^-$ and give their explicit form in terms of spinor helicity brackets.

b) Write down all photon decoupling relations for $A_4(1^-,2^-,3^+,4^+)$ and express all six amplitudes in terms of $A_4(1^-,2^-,3^+,4^+)$ and $A_4(1^-,3^+,2^-,4^+)$.

c) In addition we have the BCJ relation:

$$s_{12}\ A_4(2^-,1^-,3^+,4^+) = s_{14}\ A_4(2^-,3^+,1^-,4^+) \qquad (*) \, .$$

Check this relation by inserting the explicit expressions for the corresponding four–gluon amplitudes. Use reflection symmetry to bring $(*)$ into canonical form and use the latter to express all six amplitudes in terms of $A_4(1^-,2^-,3^+,4^+)$ only.

**2) Consider the five–gluon MHV amplitude**

$$A_5(1^-,2^-,3^+,4^+,5^+) = g_{YM}^3 \, \frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 51\rangle} \, .$$

Take the soft–limit for gluon $4^+$ and show directly (without using the general formula of the lecture):

$$A_5(1^-,2^-,3^+,4^+,5^+) \overset{p_4\to 0}{\longrightarrow} g_{YM} \, \frac{\langle 35\rangle}{\langle 34\rangle\langle 45\rangle} \, A_4(1^-,2^-,3^+,5^+) \, .$$

**3) Consider the six–gluon NMHV amplitude**

$$A_6(1^+,2^+,3^+,4^-,5^-,6^-) = g_{YM}^4 \, \Big\{ \frac{\langle 6|p_1+p_2|3]^3}{\langle 61\rangle\langle 12\rangle[34][45][5|p_1+p_6|2\rangle} \, \frac{1}{(p_6+p_1+p_2)^2} + \frac{\langle 4|p_5+p_6|1]^3}{\langle 23\rangle\langle 34\rangle[16][65][5|p_1+p_6|2\rangle} \, \frac{1}{(p_5+p_6+p_1)^2} \Big\} \, ,$$

with $\langle a|p_b+p_c|d] = \langle ab\rangle[bd] + \langle ac\rangle[cd]$.

Take the soft–limit for gluon $4^-$ and show directly (without using the general formula of the lecture):

$$A_6(1^+,2^+,3^+,4^-,5^-,6^-) \overset{p_4\to 0}{\longrightarrow} g_{YM}\ \frac{[35]}{[34][45]}\ A_5(1^+,2^+,3^+,5^-,6^-)\ .$$

**4) Consider the five–gluon MHV amplitude**

$$A_5(1^-,2^-,3^+,4^+,5^+) = g_{YM}^3\ \frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 51\rangle}\ .$$

Take the collinear–limit for gluons $4^+$ and $5^+$, i.e. $p_4 = xP$ and $p_5 = (1-x)P$ and show directly (without using the general formula of the lecture):

$$A_5(1^-,2^-,3^+,4^+,5^+) \overset{p_4\|p_5}{\longrightarrow} g_{YM}\ \frac{1}{\langle 45\rangle}\ \frac{1}{\sqrt{x(1-x)}}\ A_4(1^-,2^-,3^+,P^+)\ .$$