Exercise for Scattering Amplitude (F, T6)
Problem set 3, due to 29 May, 2019

1) The four–gluon MHV amplitude can be written as:

$$A_4(1^-,2^-,3^+,4^+) = g_{YM}^2 \; \frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle} \; .$$

Show, that the amplitude can also be written as:

$$A_4(1^-,2^-,3^+,4^+) = g_{YM}^2 \; \frac{[34]^4}{[12][23][34][41]} \quad (*) \; .$$

2) Consider the n–point amplitude

$$A_n(1^{h_1},2^{h_2},\ldots,n^{h_n}) \quad (**) \; ,$$

with particle helicities h_i.

a) Show, that if all helicities are flipped, i.e. $h_i \to -h_i$, the resulting amplitude

$$A_n(1^{-h_1},2^{-h_2},\ldots,n^{-h_n})$$

can be obtained from $(**)$ by exchaning all angle and square brackets.

b) This way rederive $(*)$ by starting at $A_4(1^+,2^+,3^-,4^-)$

3) Consider the Yukawa theory:

$$\mathcal{L} = i\bar\Psi\gamma^\mu\partial_\mu\Psi + \frac{1}{2}(\partial\phi)^2 + g\phi\Psi\bar\Psi \; .$$

If we denote the outgoing fermion by f and the outgoing anti-fermion by $\bar f$, then compute

$$A_4(\bar f^{h_1} f^{h_2} \bar f^{h_3} f^{h_4}) \; ,$$

where h_i correspond to the spin up $+\frac{1}{2}$ and spin down $-\frac{1}{2}$, respectively.

4) Consider a model with Weyl–fermion ψ and a complex scalar ϕ:

$$\mathcal{L} = i\psi\bar\sigma^\mu\partial_\mu\psi - \partial^\mu\bar\phi\partial_\mu\phi - \frac{1}{4}\lambda|\phi|^4 + (\frac{1}{2}g\phi\psi\psi + c.c) \; .$$

Show that:

$$A_4(\phi\phi\bar\phi\bar\phi) = -\lambda, \quad A_4(\phi f^- f^+ \bar\phi) = -|g|^2\frac{\langle 24\rangle}{\langle 34\rangle}, \quad A_4(f^- f^- f^+ f^+) = |g|^2\frac{\langle 12\rangle}{\langle 34\rangle} \; .$$