New Methods and Structures of Scattering Amplitudes (F, T6)
Klausur: August, 02, 2019

1) Consider Einstein–Yang–Mills amplitudes $A_{EYM}(g_1, g_2, G)$ involving two gluons and one graviton.

a) Use the little group scaling argument to write down in terms of spinor brackets all three–point amplitudes $A_{EYM}(g_1, g_2, G)$.
Hint: subject to the helicity configuration there are in total eight amplitudes $A_{EYM}(g_1^{\pm}, g_2^{\pm}, G^{\pm\pm})$ to consider.

b) Use the energy dimension argument to decide, which amplitude has to vanish ?

2) Consider the six–gluon NMHV amplitude $A_6(1^+, 2^+, 3^-, 4^+, 5^-, 6^-)$

a) How many diagrams contribute to the BCFW recursion relation following from the $[3,4\rangle$ shifts, i.e.: $\hat{\tilde{\lambda}}_3 = \tilde{\lambda}_3 - z\tilde{\lambda}_4$ and $\hat{\lambda}_4 = \lambda_4 + z\lambda_3$?

b) Is the $[3,4\rangle$ shift allowed, i.e. what can be said about behaviour of the amplitude for large z ?

c) Compute all (*or:* as much as possible of the) contributions from of the non–vanishing diagrams. In total, the amplitude becomes

$$A_6(1^+, 2^+, 3^-, 4^+, 5^-, 6^-) = g_{YM}^4 \left\{ \frac{1}{t_2} \frac{[24]^4 \langle 56 \rangle^3}{[23][34]\langle 61 \rangle \ \langle 1|2+3|4]\langle 5|3+4|2]} + \frac{1}{t_1} \frac{\langle 3|1+2|4]^4}{\langle 12 \rangle \langle 23 \rangle [45][56] \ \langle 1|2+3|4]\langle 3|1+2|6]} + \frac{1}{t_3} \frac{[12]^3 \langle 35 \rangle^4}{[61]\langle 34 \rangle \langle 45 \rangle \ \langle 5|3+4|2]\langle 3|4+5|6]} \right\}, \qquad (*)$$

with $\langle a|p_b + p_c|d] = \langle ab \rangle [bd] + \langle ac \rangle [cd]$ and $t_i = (p_i + p_{i+1} + p_{i+2})^2$.

d) Which term in $(*)$ corresponds to what diagram ?

e) What is the behaviour of $(*)$ for large z ?

3) Soft and collinear limits of $A_6(1^+, 2^+, 3^-, 4^+, 5^-, 6^-)$

a) Consider the soft–limit $p_6 \to 0$ of gluon 6^- of the amplitude $(*)$:
What does the general soft–formula yield for this limit ?
Can you extract this limit directly from $(*)$?

b) Consider the collinear–limit for gluons 5^- and 6^-, i.e. $p_5 = xP$ and $p_6 = (1-x)P$ of the amplitude $(*)$:
What does the general collinear limit–formula yield for this limit ?
Can you extract this limit directly from $(*)$?