

Disk Scattering of Open and Closed Strings

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Outline

I. Higher point open superstring amplitudes (tree)

St. St., T.R. Taylor 2006–2008.

- Universal properties and relations

II. Open & closed vs. pure open string disk amplitudes

St. St., to appear very soon.

- Sort of generalized KLT on the disk

I. Recent results for N -point open superstring amplitudes

N -point open superstring disk amplitudes in flat background

St. St., T.R. Taylor 2006–2008

Motivation: Recent results in YM in spinor basis:
compact expressions, recursion relations, . . .

- Computed N -point open superstring disk amplitude involving members of vector multiplets to all orders in α' ,
- Compact representation to all orders in α' ,
- Derived SUSY Ward identities to all orders in α'

Universal Properties

- completely model independent
- universal to all string compactifications
- any numbers of supersymmetries

Examples with members of vector multiplets

- 5-gluon MHV amplitude in superstring theory

$$A(g_1^-, g_2^-, g_3^+, g_4^+, g_5^+) = \text{Tr}(T^1 \dots T^5) (\sqrt{2} g_{YM})^3 \alpha' \\ \times \frac{\langle 12 \rangle^2}{\langle 34 \rangle^2 \langle 45 \rangle} (\langle 41 \rangle [15] \mathbf{K}_1 + \langle 42 \rangle [25] \mathbf{K}_2)$$

- Supersymmetric Ward identities in string theory

$$A(g_1^-, g_2^-, g_3^+, g_4^+, \dots, g_N^+) = \frac{\langle 12 \rangle^2}{\langle 34 \rangle^2} A(\phi_1^-, \phi_2^-, \phi_3^+, \phi_4^+, g_5^+, \dots, g_N^+)$$

- N -gluon MHV amplitude in superstring theory

$$A(g_1^-, g_2^-, g_3^+, g_4^+, \dots, g_N^+; \alpha') = \left(1 - \alpha'^2 \frac{\zeta(2)}{2} F^{(N)} \right) \\ \times A(g_1^-, g_2^-, g_3^+, g_4^+, \dots, g_N^+) + \mathcal{O}(\alpha'^3)$$

Recent results for N -point open superstring amplitudes

Note:

SUSY transformations within one multiplet (VM) using

- \mathcal{N} conserved SUSY charges Q_α^I , $I = 1, \dots, \mathcal{N}$, with $Q_\alpha^I = \oint \frac{dz}{2\pi i} V_\alpha^I(z)$
- (Space-time) SUSY transformation of open string vertex operator \mathcal{O}
on world-sheet disk $[Q^I(\eta_I) , \mathcal{O}(z)] := \oint_{C_z} \frac{dw}{2\pi i} \eta_I^\alpha V_\alpha(w) \mathcal{O}(z)$

generates SUSY Ward identities (valid to all orders in α')

c.f. also talk at Strings 2008.

Generalizations and Task

- Include chiral multiplets ($N=1$)
- Use of world-sheet supercurrent T_F
- Include closed strings to probe brane/bulk couplings

→ *Derive relations between different types of amplitudes*

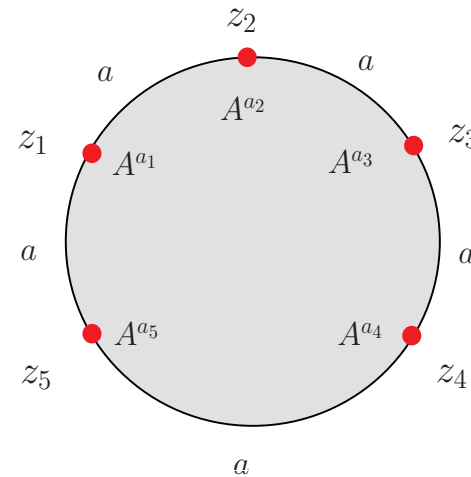
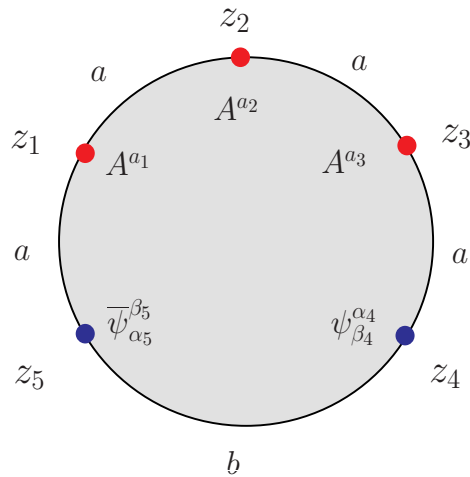
→ *Amplitudes of open and closed string moduli*

First look: N -point parton amplitudes in $D = 4$

Consider superstring disk amplitudes involving both

VM $\left\{ \begin{array}{l} g = \text{gluon} \\ \chi = \text{gaugino} \end{array} \right.$

E.g.:



CM $\left\{ \begin{array}{l} \psi = \text{fermion} \\ \phi = \text{scalar} \\ \text{in } D=4 \end{array} \right.$

$$\begin{aligned}
 A_\rho(g_1^+, g_2^-, g_3^-, g_4^-, \bar{q}_5^+) &= [V^{(5)}(s_j) - 2i P^{(5)}(s_j) \epsilon(1, 2, 3, 4)] \\
 &\times A_\rho^{FT}(g_1^+, g_2^-, g_3^-, g_4^-, \bar{q}_5^+) \\
 A_\rho(g_1^-, g_2^-, g_3^+, g_4^+, g_5^+) &= [V^{(5)}(s_j) - 2i P^{(5)}(s_j) \epsilon(1, 2, 3, 4)] \\
 &\times A_\rho^{FT}(g_1^-, g_2^-, g_3^+, g_4^+, g_5^+)
 \end{aligned}$$

Striking relation to all orders in α' !

N-point parton amplitudes in $D = 4$

$$A_{\rho}^{FT}(g_1^-, g_2^-, g_3^+, g_4^+, g_5^+) = i g_{YM}^3 \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 51 \rangle}$$

with:

$$A_{\rho}^{FT}(g_1^+, g_2^-, g_3^-, g_4^-, g_5^+) = 4 g_{YM}^3 \frac{[14][15]^3}{[12][23][34][45][51]}$$

$$V^{(5)}(s_i) = s_2 s_5 f_1 + \frac{1}{2} (s_2 s_3 + s_4 s_5 - s_1 s_2 - s_3 s_4 - s_1 s_5) f_2$$

and:

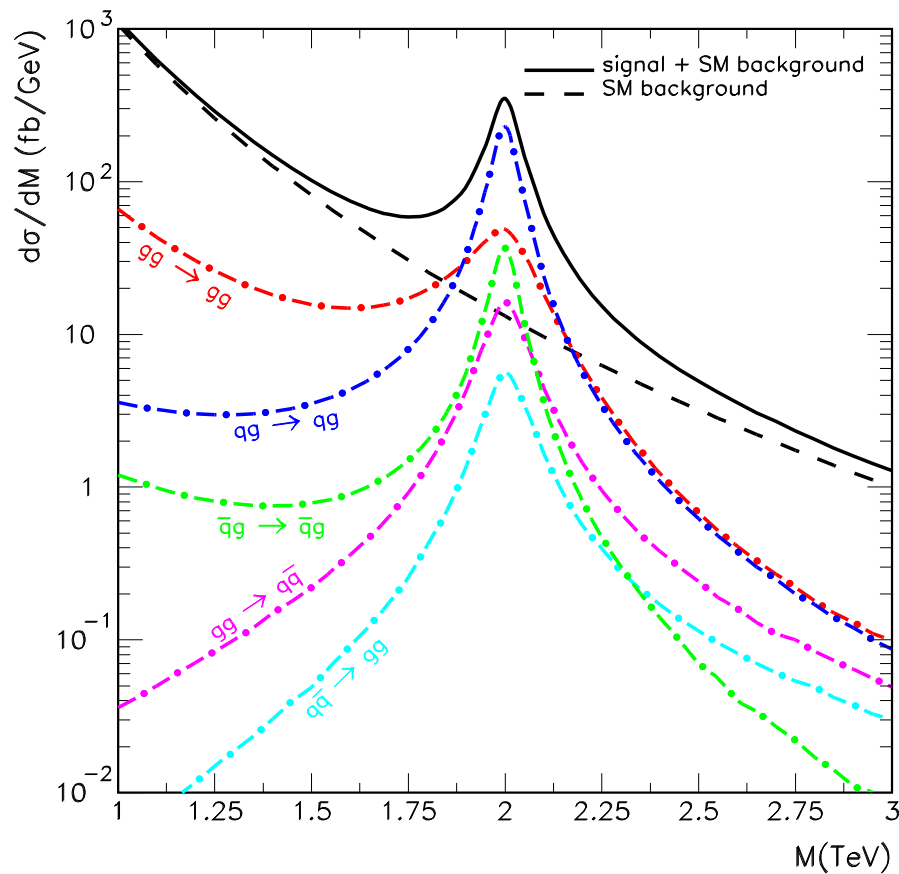
$$P^{(5)}(s_i) = f_2, \quad \epsilon(i, j, m, n) = \alpha'^2 \epsilon_{\alpha\beta\mu\nu} k_i^{\alpha} k_j^{\beta} k_m^{\mu} k_n^{\nu}$$

Relations can be generalized to:

$$\left\{ \begin{array}{l} A(g^{a_1} \dots g^{a_N}) \\ A(\chi^{a_1} \bar{\chi}^{a_2} g^{a_1} \dots g^{a_{N-2}}) \\ A(\psi^{a_1} \bar{\psi}^{a_2} g^{a_1} \dots g^{a_{N-2}}) \\ A(\phi^{a_1} \bar{\phi}^{a_2} g^{a_1} \dots g^{a_{N-2}}) \end{array} \right.$$

Amplitudes important for low string scale physics

Most relevant for signals from low string scale effects in QCD jets

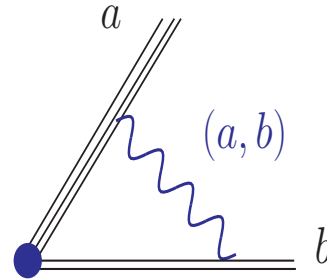


- No intermediate exchange of KKs, windings nor emission of graviton
- Useful for model-independent low-energy predictions
- Universal deviation from SM in jet distribution

Lüst, St.St., Taylor, [arXiv:0807.3333](https://arxiv.org/abs/0807.3333);
Anchordoqui, Goldberg, Nawata,
Lüst, St.St., Taylor, [arXiv:0808.0497](https://arxiv.org/abs/0808.0497);
Lüst, Schlotterer, St.St., Taylor, to appear

Appendix: Chiral matter vertex operator

Vertex operator of chiral fermion (a, b)



$$V_{\psi_{\beta}^{\alpha}}^{(-1/2)}(z, u, k) = g_{\psi} [T_{\beta}^{\alpha}]_{\alpha_1}^{\beta_1} e^{-\frac{1}{2}\phi(z)} u^{\lambda} S_{\lambda}(z) \Xi^{a\cap b}(z) e^{ik_{\rho}X^{\rho}(z)}$$

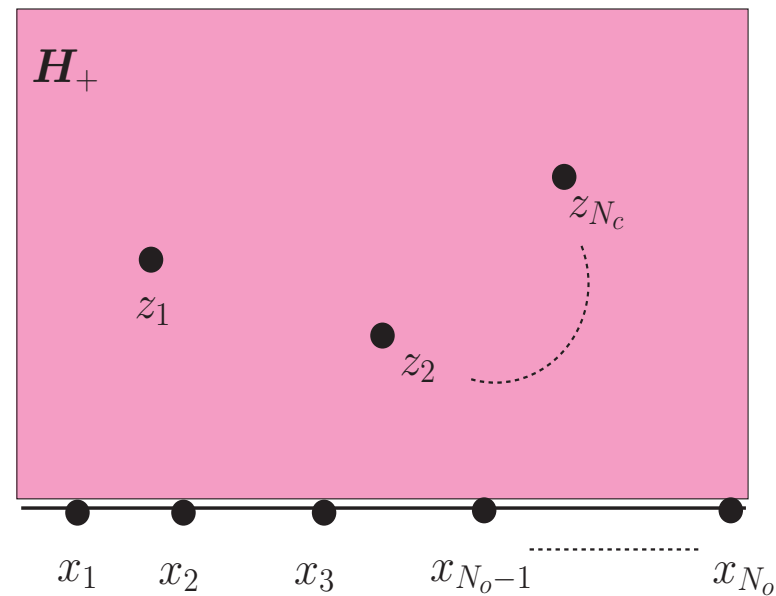
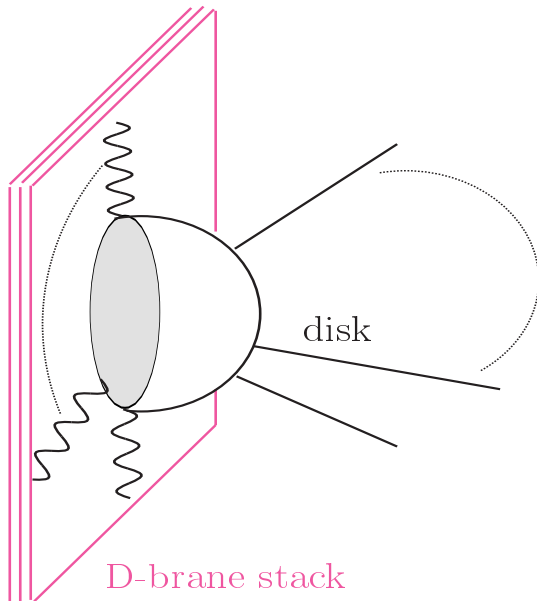
$$[g_{\psi} = (2\alpha')^{1/2} \alpha'^{1/4} e^{\phi_{10}/2}]$$

Boundary changing operator $\Xi^{a\cap b}(z)$, with $h = \frac{3}{8}$ and:

$$\langle \Xi^{a\cap b}(z_1) \Xi^{a\cap b}(z_2) \rangle = \frac{1}{(z_1 - z_2)^{3/4}}$$

II. Disk scattering of open and closed strings

$$\mathcal{A} = \sum_{\pi \in S_{N_o}/\mathbf{Z}_2} V_{\text{CKG}}^{-1} \left(\prod_{j=1}^{N_o} \int_{\mathcal{I}_\pi} dx_j \prod_{i=1}^{N_c} \int_{H_+} d^2 z_i \right) \langle \prod_{j=1}^{N_o} : V_o(x_j) : \prod_{i=1}^{N_c} : V_c(\bar{z}_i, z_i) : \rangle$$



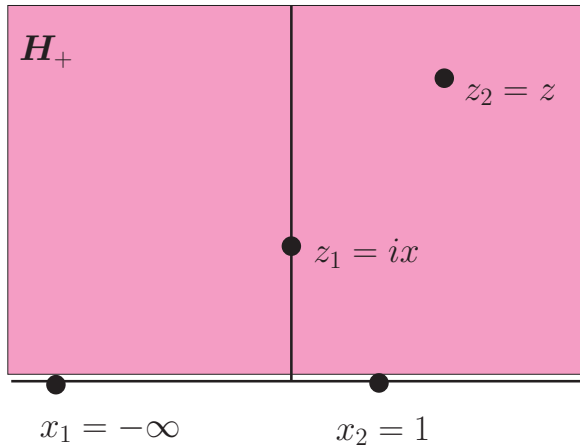
$V_o(x_i)$ = open string vertex operators inserted at x_i on the boundary of the disk

$V_c(\bar{z}_i, z_i)$ = closed string vertex operators inserted at z_i inside the disk

Example: Two open and two closed strings on the disk

With $PSL(2, \mathbf{R})$ transformation three arbitrary points $w_1, w_2 \in \mathbf{R}$ and $w_3 \in \mathbf{C}$ may be mapped to the points x_1, x_2 and z_1 :

Choice: $x_1 = -\infty$, $x_2 = 1$, $\bar{z}_1 = -ix$, $z_1 = ix$, $\bar{z}_2 = \bar{z}$, $z_2 = z$



with $z \in \mathbf{H}_+$ and $x \in \mathbf{R}^+$

$$\begin{aligned}
 \mathcal{A}(1, 2, 3, 4) &= \int_{-\infty}^{\infty} dx \langle c(-\infty)c(1)c(ix) \rangle \\
 &\times \int_{\mathbf{C}} d^2z \langle : V_o(-\infty) : : V_o(1) : : V_c(-ix, ix) : : V_c(\bar{z}, z) : \rangle
 \end{aligned}$$

Two open & two closed strings versus six open strings on the disk

- generic structure of world-sheet disk amplitude of **two open & two closed strings**:

$$\begin{aligned}
 W^{(\kappa, \alpha_0)} \begin{bmatrix} \alpha_1, \lambda_1, \gamma_1, \beta_1 \\ \alpha_2, \lambda_2, \gamma_2, \beta_2 \end{bmatrix} &= \int_{-\infty}^{\infty} dx x^{\alpha_0} (1+ix)^{\alpha_1} (1-ix)^{\alpha_2} \int_{\mathbf{C}} d^2z (1-z)^{\lambda_1} (1-\bar{z})^{\lambda_2} \\
 &\times (z-\bar{z})^{\kappa} (z-ix)^{\gamma_1} (\bar{z}-ix)^{\gamma_2} (z+ix)^{\beta_1} (\bar{z}+ix)^{\beta_2}
 \end{aligned}$$

- generic structure of world-sheet disk amplitude of **six open strings**:

$$\begin{aligned}
 F \begin{bmatrix} n_1, n_2, n_3 \\ n_4, n_5, n_6, n_7, n_8, n_9 \end{bmatrix} &= \int_0^1 dx \int_0^1 dy \int_0^1 dz x^{p_{23}+n_1} y^{p_{23}+k_{24}+p_{34}+n_2} z^{p_{16}+n_3} \\
 &\times (1-x)^{p_{34}+n_4} (1-y)^{p_{45}+n_5} (1-z)^{p_{56}+n_6} (1-xy)^{p_{35}+n_7} \\
 &\times (1-yz)^{p_{46}+n_8} (1-xyz)^{p_{36}+n_9} \quad , \quad n_i \in \mathbf{Z}
 \end{aligned}$$

Two open & two closed strings versus six open strings on the disk

After splitting the complex integral into holomorphic and anti-holomorphic pieces:
 Analytic continuation, introduce $\xi = z_1 + iz_2$, $\eta = z_1 - iz_2$, $\rho = ix$, $\rho, \xi, \eta \in \mathbf{R}$.

$$\begin{aligned}
 W^{(\kappa, \alpha_0)} \begin{bmatrix} \alpha_1, \lambda_1, \gamma_1, \beta_1 \\ \alpha_2, \lambda_2, \gamma_2, \beta_2 \end{bmatrix} &= \frac{1}{2} \int_{-\infty}^{\infty} d\rho |\rho|^{\alpha_0} |1 + \rho|^{\alpha_1} |1 - \rho|^{\alpha_2} \\
 &\times \int_{-\infty}^{\infty} d\xi \int_{-\infty}^{\infty} d\eta |1 - \xi|^{\lambda_1} |\xi - \rho|^{\gamma_1} |\xi + \rho|^{\beta_1} \\
 &\times |1 - \eta|^{\lambda_2} |\eta - \rho|^{\gamma_2} |\eta + \rho|^{\beta_2} |\xi - \eta|^{\kappa} \Pi(\rho, \xi, \eta)
 \end{aligned}$$

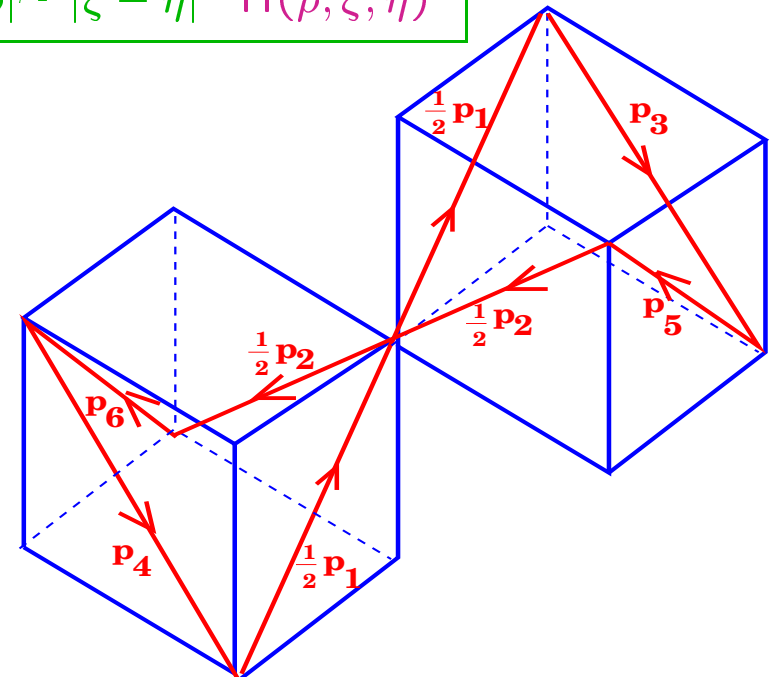
Answer: Six open strings, with:

$$z_1 = -\infty, \quad z_2 = 1, \quad z_3 = -\rho,$$

$$z_4 = \rho, \quad z_5 = \xi, \quad z_6 = \eta$$

$$p_1 = k_1, \quad p_2 = k_2,$$

$$p_3 = p_4 = \frac{1}{2}k_3, \quad p_5 = p_6 = \frac{1}{2}k_4$$



Two open & two closed strings versus six open strings on the disk

$$W^{(\kappa, \alpha_0)} \begin{bmatrix} \alpha_1, \lambda_1, \gamma_1, \beta_1 \\ \alpha_2, \lambda_2, \gamma_2, \beta_2 \end{bmatrix} = \sigma_\gamma \sin(\pi\beta_2) (I_{163542} + I_{163524} + I_{164532}) \\ + \sin(\pi\lambda_2) (I_{134526} + I_{143526}) + \sigma_\lambda \sigma_\gamma \sin(\pi\gamma_2) I_{132546} + R$$

Six I 's are six-open string amplitudes $F[\dots]$

with the six open string orderings

$$\left\{ \begin{array}{l} I_{163542} : z_1 < z_6 < z_3 < z_5 < z_4 < z_2 \\ I_{163524} : z_1 < z_6 < z_3 < z_5 < z_2 < z_4 \\ I_{134526} : z_1 < z_3 < z_4 < z_5 < z_2 < z_6 \\ I_{132546} : z_1 < z_3 < z_2 < z_5 < z_4 < z_6 \\ I_{164532} : z_1 < z_6 < z_4 < z_5 < z_3 < z_2 \\ I_{143526} : z_1 < z_4 < z_3 < z_5 < z_2 < z_6 \end{array} \right.$$

Appendix

To obtain canonical form of open string amplitudes given by generalized Euler integrals (along segment $[0, 1]$)

requires transformations:

$$\begin{aligned} I_1 : \quad \rho &\rightarrow -1 + \frac{2}{1 + yz}, & \xi &\rightarrow 1 - \frac{2y}{1 + yz}, & \eta &\rightarrow 1 - \frac{2}{x(1 + yz)} \\ I_2 : \quad \rho &\rightarrow \frac{1}{1 - 2yz}, & \xi &\rightarrow \frac{1 - 2y}{1 - 2yz}, & \eta &\rightarrow -\frac{2 - x}{x(1 - 2yz)} \\ I_3 : \quad \rho &\rightarrow \frac{xy}{2 - xy}, & \xi &\rightarrow \frac{(2 - x)y}{2 - xy}, & \eta &\rightarrow \frac{2 - xyz}{z(2 - xy)} \\ I_4 : \quad \rho &\rightarrow -\frac{1}{1 - 2xy}, & \xi &\rightarrow \frac{1 - 2y}{1 - 2xy}, & \eta &\rightarrow -\frac{2 - z}{z(1 - 2xy)} \end{aligned}$$

Open & closed vs. pure open string disk amplitude

General: Disk amplitude involving N_o open and N_c closed strings is mapped to disk amplitudes of $N_o + 2N_c$ open strings

E.g.:

$$\begin{array}{lll} N_o = 2, N_c = 1 & \implies & \text{four open strings} \\ N_o = 3, N_c = 1 & \implies & \text{five open strings} \\ N_o = 4, N_c = 1, N_o = 2, N_c = 2 & \implies & \text{six open strings} \\ & \vdots & \vdots \end{array}$$

E.g.:

$$\begin{array}{l} N_o = 2, N_c = 1 : G[\alpha_0, \alpha_1, \alpha_2] = \sin(\pi\lambda) I_{1234} \\ N_o = 3, N_c = 1 : G^{(\alpha)} \begin{bmatrix} \lambda_1, \gamma_1 \\ \lambda_2, \gamma_2 \end{bmatrix} = \sin(\pi\lambda_2) I_{15243} + \sigma_\gamma \sin(\pi\alpha) I_{12345} \end{array}$$

Plahte identities: Analysis of contour integrals in the complex plane yield $(N_o + 2N_c - 3)!$ -dimensional basis of functions

Open & closed vs. pure open string disk amplitudes

Sort of generalized KLT on the disk

$$V_{\text{closed}}(\bar{z}_i, z_i) \simeq V_{\text{open}}(\bar{z}_i) V_{\text{open}}(z_i) \simeq V_{\text{open}}(\eta_i) V_{\text{open}}(\xi_i)$$

$z_i \in \mathbf{C}$ $\eta_i, \xi_i \in \mathbf{R}$

E.g.:

$$\begin{aligned} & \langle A_{\mu_1}(x_1) A_{\mu_2}(x_2) G_{\mu_3\mu_4}(\bar{z}_1, z_1) G_{\mu_5\mu_6}(\bar{z}_2, z_2) \rangle \\ & \simeq \langle A_{\mu_1}(x_1) A_{\mu_2}(x_2) A_{\mu_3}(\eta_1) A_{\mu_4}(\xi_1) A_{\mu_5}(\eta_2) A_{\mu_6}(\xi_2) \rangle \\ & \langle A_{\mu_1}(x_1) A_{\mu_2}(x_2) F_{\alpha\beta}(\bar{z}_1, z_1) F_{\gamma\delta}(\bar{z}_2, z_2) \rangle \\ & \simeq \langle A_{\mu_1}(x_1) A_{\mu_2}(x_2) \chi_\alpha(\eta_1) \chi_\beta(\xi_1) \chi_\gamma(\eta_2) \chi_\delta(\xi_2) \rangle \end{aligned}$$

Open & closed vs. pure open string disk amplitudes

This map reveals
important relations between
open & closed string disk amplitudes
and pure open string disk amplitudes !