The hidden sector becomes scale invariant at \( \frac{1}{10^3} \) couplings. Then become gies above \( \frac{1}{10^3} \) below which the sector exhibits scale invariance. At energy beyond the standard model. In this scenario, there is a constant. Because of scale invariance, the phase space for \( \Omega \) of \( \Omega \), a hidden-sector operator \( O_{\text{UV}} \) of dimension \( d_{\text{UV}} \) couples to standard-model operators \( O_{\text{SM}} \) of dimension \( n \) via the exchange of heavy particles of mass \( M \),

\[
\mathcal{L}_{\text{UV}} = \frac{O_{\text{UV}} O_{\text{SM}}}{M^{d_{\text{UV}} + n - 4}}. \tag{1}
\]

The hidden sector becomes scale invariant at \( \Lambda \). The couplings then become

\[
\mathcal{L}_U = C_{U} \frac{\Lambda^{d_{\text{UV}} - d}}{M^{d_{\text{UV}} + n - 4}} O_{\text{SM}} O_{U}, \tag{2}
\]

where \( O_{U} \) is the unparticle operator of dimension \( d \) in the low-energy limit, and \( C_{U} \) is a dimensionless coupling constant. Because of scale invariance, the phase space for \( O_{U} \) resembles that of \( d \) massless particles; the salient feature of the unparticle sector is that \( d \) may take on non-integer values. Unparticle phenomenology has been investigated in a large number of recent papers [3–45].

If the unparticle sector indeed appears at low energies in the form of new massless fields coupled very weakly to standard-model particles, one expects that the usual stellar energy-loss limits \( \frac{1}{10^3} \) are typical examples where astrophysical limits preclude any realistic hope of finding these particles directly in collider experiments.

If a scalar unparticle operator of dimension \( d < 2 \) couples to the standard-model Higgs, then scale invariance in the hidden sector is broken once the Higgs acquires a nonzero VEV so that there is no unparticle signature at low energies [12]. In this case the SN 1987A energy-loss argument is moot. Therefore, the astrophysical limit serves to constrain the possible realizations of the unparticle concept that could show up as missing energy at colliders.

The dominant lowest-order process for the emission of new particles \( X \) in a SN core tends to be nucleon bremsstrahlung \( N + N \rightarrow N + N + X \). The nucleon density is huge. Their interaction rate is large, and this process involves the small coupling of the new particles to lowest order. The emission of unparticles will be fully analogous, apart from phase-space modifications. In addition, processes of the form \( f + f \rightarrow U \) now have a non-vanishing phase space and provide additional contributions. Therefore, the SN 1987A energy-loss argument provides very restrictive constraints on the unparticle couplings to nucleons, neutrinos, electrons and muons.

Numerical studies of SN energy losses by axions or Kaluza-Klein gravitons reveal that the neutrino burst would have been excessively shortened unless the volume energy-loss rate of the SN core obeys [46,47]

\[
Q \leq 3 \times 10^{33} \text{ erg cm}^{-3} \text{ s}^{-1}, \tag{3}
\]

where \( Q \) is to be calculated at the benchmark conditions \( T = 30 \text{ MeV} \) and \( \rho = 3 \times 10^{14} \text{ g cm}^{-3} \). Armed with this simple criterion, all that is needed is an estimate of \( Q \) for unparticle emission.

While this is not hard, and while the case of nucleon bremsstrahlung emission was already briefly discussed in the unparticle literature [14], the exploding interest in the unparticle idea justifies working out this argument and its consequences in some detail.\(^1\)

### II. NUCLEONS

In the simplest case unparticles couple via a vector current to fermions, an assumption made in all previous studies. Beginning with nucleons, the structure of the

\(^1\) Another study of stellar energy-loss constraints [45] has become available at the time of revising our manuscript. The conclusions are similar where our works overlap.
coupling is

$$\mathcal{L}_{\text{UN}} = C_{\text{UN}} \frac{\Lambda_{\text{UV}}^{d-3}}{M_{\text{UV}}^{d-1}} \vec{N} \gamma \mu N \nu_U^\mu.$$

(4)

Motivated by the cases of axions and Kaluza-Klein gravitons we expect bremsstrahlung of the form $N + N \rightarrow N + N + U$ to be the dominant emission process.

In order to estimate the emission rate we note that for the assumed vector coupling this process would seem to be normal dipole emission similar to electromagnetic bremsstrahlung. However, if the interaction with all nucleons is the same, bremsstrahlung is suppressed in the nonrelativistic limit, just as it is suppressed in the nonrelativistic limit for $e^- + e^- \rightarrow e^- + e^- + \gamma$ because for two equal particles the center of mass and the center of charge coincide, preventing dipole radiation in a collision. Therefore, we expect quadrupole radiation to dominate. Nonrelativistic dipole emission in a collision would involve a factor $(\Delta \nu)^2$ where $\Delta \nu$ is the velocity change of the radiating particle in the collision, whereas in the quadrupole case we expect $(\Delta \nu)^4$ in analogy to graviton bremsstrahlung that was studied in the context of Kaluza-Klein graviton emission [54–56].

Assuming that the $NN$ collision is approximately isotropic, the change of velocity in a collision is similar to the velocity itself. Therefore, we expect a factor $v^4$ in the rate, corresponding to a scaling $(T/m)^2$ in the emission rate from a thermal medium. Further the bremsstrahlung rate is proportional to a thermal average of the ordinary scattering rate $\sigma \nu n_B^2$ where $\sigma$ the $NN$ collision cross section and $n_B$ the nucleon (baryon) density. The energy emitted in a single collision is of order $T$. Collecting all factors we find on dimensional grounds that the energy-loss rate is

$$Q = C_{\text{UN}} \frac{\Lambda_{\text{UV}}^{2(d-3)}}{M_{\text{UV}}^{d-1}} T^{2d-2} \sigma \nu n_B^2 \frac{T^7}{m^{3/2}}.$$  

(5)

A recent more detailed calculation finds the same scaling, but a somewhat smaller numerical factor [45]. We note, however, that a full calculation requires modeling the nucleon-nucleon interaction potential in a dense nuclear medium, a feat that has not been achieved in any study of supernova particle emission.

To evaluate Eq. (5) this rate we assume $\sigma = 25 \times 10^{-27}$ cm$^2$ that is typical for the relevant conditions and was recommended in the context of Kaluza-Klein graviton emission [55]. Applying Eq. (3) we find the constraint

$$C_{\text{UN}} \frac{\Lambda_{\text{UV}}^{d-3}}{M_{\text{UV}}^{d-1}} (30 \text{ MeV})^{d-1} \lesssim 1.2 \times 10^{-10}.$$  

(6)

Assuming $d_{\text{UV}} = 3$ and $M = 1000$ TeV, we find

$$\Lambda \lesssim \begin{cases} 
10 \text{ GeV} & d = 1, \\
80 \text{ GeV} & d = 3/2, \\
4 \text{ TeV} & d = 2, 
\end{cases}$$  

(7)

where we have set $C_{\text{UN}} = 1$ for simplicity. These bounds are somewhat less restrictive than those derived in Ref. [14] from scaling the usual axion limits to the unparticle case. (Such a scaling is not trivial because axion emission is caused by nucleon spin flips in collisions due to the axial-current nature of the interaction, in contrast to the quadrupole emission for a vector interaction.)

Figure 1 shows the same constraints on the $(\Lambda, M)$ plane. These should be compared with analogous bounds from collider experiments in, e.g., Fig. 6 of Ref. [23]. The SN limits are always far more restrictive. As an illustration, for $\Lambda < 1000$ GeV, Ref. [23] quotes the constraints $M > 7500$ GeV ($d = 1$), $M > 2500$ GeV ($d = 3/2$), and $M > 1000$ GeV ($d = 2$), to be compared with our much more severe SN limits $M > 10^8$ GeV ($d = 1$), $M > 7 \times 10^9$ GeV ($d = 3/2$), and $M > 5 \times 10^9$ GeV ($d = 2$). Note here that the bound in [23] is derived using the unparticle coupling to leptons whereas our SN bound comes from the nucleon-unparticle coupling. However, as will be seen in the next section the supernova energy-loss argument can also be used to constrain the unparticle-lepton coupling and to derive a constraint which is comparable to the one derived here on the unparticle-nucleon interaction.

Besides nucleon bremsstrahlung we have also considered the Compton process $\gamma + p \rightarrow p + U$. From textbook expressions for the Compton cross section for $d = 1$ and assuming a proton fraction of 0.3, we infer a limit corresponding to Eq. (6) with a right-hand side of $3 \times 10^{-9}$. While this is considerably less restrictive, it still excludes possible collider signatures.

The SN 1987A argument is not necessarily the most restrictive limit on the unparticle-nucleon interaction. Another stellar energy-loss argument is based on the

![FIG. 1 (color online). Constraints on vector unparticle operators from SN bremsstrahlung emission, assuming $d_{\text{UV}} = 3$, for $d = 1, 3/2,$ and 2 as indicated. The regions below the contours are excluded.](121701(R) (2007))
helium-burning lifetime of horizontal branch stars in globular clusters. A novel energy-loss channel should not exceed about 10 erg g\(^{-1}\) s\(^{-1}\) at a density of about \(10^4\) g cm\(^{-3}\) and a temperature of about 10\(^8\) K = 8.6 keV. The medium consists primarily of helium, carbon, and oxygen. Using the Compton process more restrictive, the SN bound quickly wins for next order the process bremsstrahlung process indeed provides the most restrictively small.

Because of its much larger temperature. Moreover, depending on the isospin structure of the unparticle couplings to nucleons, the interaction with \(f\) because of energy-momentum conservation. At \(T = 30\) MeV the number density of muons is suppressed only by a factor \(~2.5\) relative to massless fermions, always ignoring chemical potentials that are small for muons in a SN core. An additional suppression factor of about 0.5 comes from the relative velocity of muons so that the annihilation rate overall is roughly 0.2 times that of a massless fermion species.

Overall, then, the SN 1987A argument based on the bremsstrahlung process indeed provides the most restrictive limit on the unparticle interactions with nucleons.

III. CHARGED LEPTONS AND NEUTRINOS

For massless particles \(X\), the process \(\bar{f} + f \rightarrow X\) is forbidden because of energy-momentum conservation. At next order the process \(\bar{f} + f \rightarrow X + X\) is highly suppressed because of the additional power of the coupling constant. For unparticles, however, the process \(\bar{f} + f \rightarrow U\) is allowed because of the nonstandard final state phase space and can be a dominant energy-loss process in a SN core.

The emissivity from pair annihilation of neutrinos is roughly estimated by

\[ Q \sim C_{\nu_f} \frac{\Lambda^{2(d_{\nu_f} - d)}}{M^{2(d_{\nu_f} - 1)}} T^{2d + 3}, \]

which leads to the constraints, for \(C_{\nu_f} = 1\),

\[ \Lambda \leq \begin{cases} - & d = 1 \\ 20 \text{ GeV} & d = 3/2 \\ 500 \text{ GeV} & d = 2 \end{cases} \]

assuming \(d_{\nu_f} = 3\) and \(M = 1000\) TeV as before.

For \(d = 3/2\) and 2 these bounds are almost identical to those deduced from bremsstrahlung in Eq. (7), although they are based on different physics. The reason is that the smaller neutrino density is compensated by the fact that the \(\nu \bar{\nu}\) channel is not suppressed by the \(\nu^4\) factor. For different values of \(M\) the bounds on \(\Lambda\) scale in the same way as those from the bremsstrahlung calculation so that the results of Fig. 1 apply also in this case. No bound exists for \(d = 1\) from \(\bar{f} + f \rightarrow U\), since this case corresponds to annihilation into a single massless particle, and its rate must vanish exactly because of energy-momentum conservation.

The bounds Eq. (9) were calculated assuming nondegenerate, relativistic fermions. This approximation is very good for \(\nu_\mu\) and \(\nu_\tau\). However, electron neutrinos are degenerate with a chemical potential \(\mu\) typically of order \(150–200\) MeV, implying a degeneracy parameter around 5–7 for \(T = 30\) MeV. The presence of a chemical potential reduces the number of pairs, i.e., the quantity \(n_\mu n_\nu\) is largest for a vanishing chemical potential. However, for degeneracy factors up to roughly 8 the suppression of the annihilation rate is less than an order of magnitude (for illustration see Fig. 2 of Ref. [57]). The degeneracy factor for electrons is only slightly larger so that the suppression of \(e^+ e^-\) annihilation is only slightly worse.

Because of its high temperature the hot proto-neutron star is also abundant in muons. The muon rest mass of 106 MeV does not lead to a very substantial difference in number density relative to massless fermions. At \(T = 30\) MeV the number density of muons is suppressed only by a factor \(~2.5\) relative to massless fermions, always ignoring chemical potentials that are small for muons in a SN core. An additional suppression factor of about 0.5 comes from the relative velocity of muons so that the annihilation rate overall is roughly 0.2 times that of a massless fermion species.

Last, we note that a bound on the \(d = 1\) scenario can still be obtained from pair annihilation of charged leptons based on the process \(e^+ + e^- \rightarrow U + \gamma\) or \(\mu^+ + \mu^- \rightarrow U + \gamma\) which is not phase-space suppressed. Since the squared matrix element is suppressed only by \(O(\alpha)\) relative to \(\bar{f} + f \rightarrow U\), the bound for \(d = 1\) can be estimated from taking the emissivity as \(\alpha\) times the naive rate given in Eq. (8), and taking into account the suppression from degeneracy (electrons) or the mass threshold (muons). Note that for \(d = 1\) the suppression factors enter as a fourth root, giving an estimated bound for \(d = 1\) of \(\Lambda \leq 30–40\) GeV for \(M = 1000\) TeV.

Given the overall numerical uncertainties of our limits, we conclude that the pair annihilation and bremsstrahlung limits are nearly identical for all cases. Apart from possible difference in the overall coefficients \(C_{\nu_f}\) to different particle species, the interaction with nucleons, neutrinos of all flavors, electrons and muons are all constrained by the limits shown in Fig. 1.

IV. CONCLUSIONS

We have applied the well-known energy-loss limit based on the SN 1987A neutrino burst duration to Georgi’s new idea of unparticles that can manifest themselves as missing energy in collider experiments with a peculiar phase-space behavior. As expected, the SN limits are very restrictive as long as the unparticle radiation can be emitted without threshold at the relatively low energies prevalent in the SN context. Our approximate constraints shown in Fig. 1 apply without significant modifications to nucleons, neutrinos of all flavors, electrons and muons.

For all of these particles except electrons, the SN 1987A constraints are the most restrictive. For electrons, usually much more restrictive limits on their interactions with axions, scalar or vector bosons derive from the energy-
loss argument applied to globular cluster stars or white dwarfs [46–48]. The same is true for unparticles as shown recently in Ref. [45]. These authors also show that constraints on deviations from the equivalence principle and fifth-force limits provide very restrictive limits for $d$ near 1 where unparticles mediate long-range forces.

Without specific models it is unclear, however, how unparticles couple to different particle species. Therefore, we believe the SN 1987A constraint is extremely useful because it covers a broad range of particle species with almost identical limits. The SN 1987A argument is unique for neutrinos and muons, and the most restrictive constraint for nucleons.

Unparticle signatures can still be detected at colliders in models where scale invariance in the hidden sector is broken by the Higgs vacuum expectation value. In this case the SN emission is suppressed by threshold effects. Thus our astrophysical limits provide a severe restriction on the type of unparticle models that can be detected at colliders.

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UNPARTICLE CONSTRAINTS FROM SUPERNOVA 1987A


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