Analysis of the SN 1987A neutrinos with a flexible spectral shape

Alessandro Mirizzi1,2 and Georg G. Raffelt1

1Max-Planck-Institut für Physik (Werner-Heisenberg-Institut), Föhringer Ring 6, 80805 München, Germany
2Dipartimento di Fisica and Sezione INFN di Bari, Via Amendola 173, 70126 Bari, Italy

(Received 27 July 2005; published 8 September 2005)

We analyze the neutrino events from the supernova (SN) 1987A detected by the Kamiokande-II (KII) and Irvine-Michigan-Brookhaven (IMB) experiments. For the time-integrated flux we assume a quasi-thermal spectrum of the form $E/E_\text{th}^\alpha e^{-\alpha E/E_\text{th}}$ where $\alpha$ plays the role of a spectral index. This simple representation not only allows one to fit the total energy $E_\text{tot}$ emitted in $\bar{\nu}_e$ and the average energy $(E_\bar{\nu}_e)$, but also accommodates a wide range of shapes, notably antipinched spectra that are broader than a thermal distribution. We find that the pile-up of low-energy events near threshold in KII forces the best-fit value for $\alpha$ to the lowest value of any assumed prior range. This applies to the KII events alone as well as to a common analysis of the two data sets. The preference of the data for an “unphysical” spectral shape implies that one can extract meaningful values for $(E_\bar{\nu}_e)$ and $E_\text{tot}$ only if one fixes a prior value for $\alpha$. The tension between the KII and IMB data sets and theoretical expectations for $(E_\bar{\nu}_e)$ is not resolved by an antipinched spectrum.

DOI: 10.1103/PhysRevD.72.063001

PACS numbers: 14.60.Pq, 95.55.Vj, 95.85.Ry, 97.60.Bw

I. INTRODUCTION

The neutrino observations of supernova (SN) 1987A [1–6] have long been taken as a confirmation of the salient features of our physical understanding of the core-collapse SN phenomenon. At the same time, a detailed interpretation of the data is difficult because of a number of “anomalies” [15]. In particular, the energy spectra implied by the Kamiokande-II (KII) and Irvine-Michigan-Brookhaven (IMB) events are barely consistent with each other. Moreover, the average $\bar{\nu}_e$ energy implied by KII is much lower than expected from numerical simulations. The energies implied by IMB alone would be compatible with theoretical models, but the results of a common KII and IMB analysis are difficult to square with expectations.

In the presence of neutrino oscillations, the $\bar{\nu}_e$ flux observed in a detector is a superposition of the $\bar{\nu}_e$ and $\bar{\nu}_\mu,\tau$ fluxes produced at the source. Therefore, the KII and IMB detectors may have observed different fluxes because the SN 1987A neutrinos have traversed different sections of the Earth so that matter effects can modify the observed superposition of source spectra [16,17]. However, it is expected that $\langle E_{\bar{\nu}_e} \rangle > \langle E_{\bar{\nu}_\mu,\tau} \rangle$ so that oscillations aggravate the tension between observed and expected $\bar{\nu}_e$ energies [11,16,17]. In any event, the differences between $\langle E_{\bar{\nu}_e} \rangle$ and $\langle E_{\bar{\nu}_\mu,\tau} \rangle$ are probably much smaller than had been thought previously [18–20] so that the Earth matter effect is no longer expected to cause gross modifications of the observed $\bar{\nu}_e$ spectrum. Of course, the relatively subtle modifications caused by Earth matter effects can be crucial for identifying the neutrino mass ordering from the high-statistics neutrino signal of a future galactic SN [21–30].

Previous studies of the SN 1987A neutrinos usually assumed a thermal spectrum and then extracted $\langle E_{\bar{\nu}_e} \rangle$ and the overall flux from the individual detector signals or from a combined analysis. One exception is the analysis of Janka and Hillebrandt [9] who assumed an effective Fermi-Dirac distribution of the form

$$\varphi(E) \propto \frac{E^2}{e^{E/T-\eta}+1},$$

where $T$ is an effective temperature and $\eta$ a degeneracy parameter. In their maximum-likelihood analysis they allowed only for positive values of $\eta$. With this prior they found a best-fit value of $\eta = 0$ for both the KII and IMB data sets, suggesting that the data prefer the broadest allowed distribution compatible with the prior range for $\eta$. However, even allowing for negative values for $\eta$ would not have changed these results because $\eta \rightarrow -\infty$ corresponds to a Maxwell-Boltzmann spectrum, differing only marginally from the Fermi-Dirac case with $\eta = 0$.

The main purpose of our new study is to investigate if better internal agreement of the SN 1987A data as well as better agreement between the data and theoretical expectations can be achieved if a more flexible representation of the spectral shape is assumed. Numerical studies of neutrino transport suggest that the instantaneous spectra are “pinched,” i.e. that they are narrower than a thermal spectrum [9,19]. A pinched spectrum can be represented by a Fermi-Dirac distribution with positive $\eta$. However, the SN 1987A data measure the time-integrated spectrum, i.e. a superposition of instantaneous spectra with varying average energies. Therefore, the integrated spectrum may well be broader, not narrower, than a thermal spectrum, i.e. it may well be antipinched.

If the time-integrated spectrum is quasithermal in the sense that it rises from zero for low energies, reaches a maximum, and has a long tail to high energies, then the simplest conceivable representation is [19]...
\[
\varphi(E) = \frac{1}{E_0} \frac{(\alpha + 1)^{\alpha+1}}{\Gamma(\alpha + 1)} \frac{E^\alpha}{E_0^\alpha} \exp \left[-(\alpha + 1) \frac{E}{E_0}\right], 
\] (2)

where \( \int \varphi(E)dE = 1 \) and \( E_0 \) is an energy scale with the property \( \langle E \rangle = E_0 \). The numerical parameter \( \alpha \) controls the width of the distribution,

\[
\frac{\langle E^2 \rangle - \langle E \rangle^2}{\langle E \rangle^2} = \frac{1}{1 + \alpha}.
\] (3)

We note that \( \alpha = 2 \) corresponds to a Maxwell-Boltzmann spectrum, \( \alpha > 2 \) to a pinched spectrum with suppressed high- and low-energy tails, and \( \alpha < 2 \) to an antipinched spectrum. For \( \alpha \to \infty \) the spectrum becomes \( \delta(E - E_0) \).

Since the oscillations effect on the detected \( \bar{\nu}_e \) spectrum is reasonably small (see Sec. I), it makes sense to fit directly the effective \( \bar{\nu}_e \) spectrum, after the oscillations, with the distribution of Eq. (2).

In Sec. II we present the SN 1987A neutrino data and perform a new maximum-likelihood analysis. A summary and conclusions are given in Sec. III.

II. MAXIMUM-LIKELIHOOD ANALYSIS

A. SN 1987A Data

We limit our analysis to the SN 1987A data of the KII [1,2] and IMB [3,4] detectors that measure SN neutrinos almost exclusively by the inverse beta reaction \( \bar{\nu}_e + p \to n + e^+ \). We show the measured positron spectra in Fig. 1 where we have left out the KII event No. 6 that is attributed to background. In Table I we summarize the properties of the neutrino signal in the two detectors. We do not include the Baksan Scintillator Telescope (BST) data [5,6] because it is much more uncertain which of the events have to be attributed to background, i.e. in a maximum-likelihood analysis one would have to model the background. This requires to use the time structure of the neutrino burst [13,14], whereas we limit our study to the time-integrated flux.

B. Maxwell-Boltzmann Spectrum

We perform a maximum-likelihood analysis of the SN 1987A signal along the lines of the previous literature such as Ref. [11]. For the detection cross section we use an updated analytic fit [31] and for the detection efficiencies we use the analytic fit functions of Ref. [32].

In order to compare with the previous literature we first consider a Maxwell-Boltzmann \( \bar{\nu}_e \) spectrum, i.e. we use \( \alpha = 2 \) in Eq. (2). As fit parameters we use the average \( \bar{\nu}_e \) energy \( E_0 \) and the total energy \( E_{tot} \) emitted by SN 1987A in the form of \( \bar{\nu}_e \). Of course, these parameters refer to the spectrum measured in the detectors after the partial flavor swapping caused by neutrino oscillations. Our results shown in Fig. 2 agree with the previous literature and illustrate once more the tension between the average energies implied by the two detectors.

C. General Spectrum

Next we use a general spectrum of the form Eq. (2) with \( E_0 \), \( E_{tot} \), and \( \alpha \) as our fit parameters with a prior \( \alpha \geq 0 \). We show the best-fit values for these parameters as well as the implied event numbers and average positron energies in the detectors in Table II. This analysis is performed for each detector separately as well as for the joint case.

The IMB data alone prefer a functional form with a very large value for \( \alpha \), i.e. essentially a \( \delta \) function. This behavior is intuitively obvious because 5 of the 8 IMB events have positron energies in the very narrow range 36–39 MeV. The KII data alone, on the other hand, prefer \( \alpha = 0 \), i.e. a huge total flux of low-energy \( \bar{\nu}_e \). The pileup of

<table>
<thead>
<tr>
<th>Detector</th>
<th>( N_{events} )</th>
<th>( \langle E_{e^+} \rangle ) [MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>KII</td>
<td>11</td>
<td>15.4 ± 1.1</td>
</tr>
<tr>
<td>IMB</td>
<td>8</td>
<td>31.9 ± 2.3</td>
</tr>
</tbody>
</table>
events just above the KII threshold of 7.5 MeV implies that the turnover at low energies of a quasithermal spectrum is not visible in the data, which are sensitive only to the decreasing higher-energy tail of the spectrum. This feature is reproduced in the likelihood by pushing \( E_0 \) toward low values and broadening as much as possible the width. Moreover, the low value of \( E_0 \) is compensated by a huge amount of the total emitted energy \( E_{\text{tot}} \). We have checked that even when we allow for negative values the maximum of the likelihood always coincides with the lowest allowed value. Of course, for \( \alpha \leq -1 \) the total neutrino flux diverges.

Contrary to our expectation, this behavior remains the same for a joint analysis of the two data sets, even though the total flux is smaller and the average \( E_\nu \) energy is larger.

In Fig. 3 we show the difference \( \Delta \ln \mathcal{L} \) relative to the best-fit value as a function of \( \alpha \) for KII (dotted line), IMB (dashed line), and for the joint data set (continuous line). The 95% C.L. for \( \alpha \) is indicated with the dot-dashed horizontal line. We have marginalized over \( E_0 \) and \( E_{\text{tot}} \).

D. Fixed Prior Values for \( \alpha \)

We conclude that in order to extract information on the average \( \bar{E}_\nu \) energy and the total flux it is not useful to marginalize over the parameters \( E_0 \) and \( E_{\text{tot}} \). The 95% C.L. allowed range for a single degree of freedom corresponds to \( \Delta \ln \mathcal{L} = 1.92 \) shown as a horizontal dot-dashed line in Fig. 3. Evidently IMB alone has no strong preference for any spectral shape. The 95% upper limit on \( \alpha \) is lower for the combined data than for KII alone because the combined data also prefer a larger \( E_0 \) and lower total flux. Overall, the data do not distinguish in a useful way between different plausible spectral shapes. In particular, the data do not prefer a quasithermal spectrum but rather a monotonically falling one, in contrast with plausible expectations.

### Table II

<table>
<thead>
<tr>
<th>Data Set</th>
<th>( E_{\text{tot}} ) (x10^{52} \text{ erg})</th>
<th>( E_0 ) (MeV)</th>
<th>( \alpha )</th>
<th>( N_{\text{events}} (E_{\nu}) )</th>
<th>( N_{\text{events}} (E_{\nu}^*) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>KII</td>
<td>17.4</td>
<td>3.7</td>
<td>0.0</td>
<td>10.9</td>
<td>15.3</td>
</tr>
<tr>
<td>IMB</td>
<td>1.1</td>
<td>30.6</td>
<td>60.0</td>
<td>6.6</td>
<td>30.3</td>
</tr>
<tr>
<td>Joint</td>
<td>10.1</td>
<td>5.4</td>
<td>0.0</td>
<td>14.1</td>
<td>19.2</td>
</tr>
</tbody>
</table>

The implied characteristics of the expected detector signals are also shown.
III. DISCUSSION AND SUMMARY

The expected neutrino energies from core-collapse supernovae (SNe) are of practical importance for sensitivity forecasts of a future galactic SN neutrino detection, in particular, with regard to neutrino mixing parameters [21–30]. The expected energies are also important in the context of current limits and future detection possibilities of the diffuse SN neutrino background from all SNe in the universe [33–37]. The internal tension within the SN 1987A neutrino data as well as the tension with theoretical expectations has persuaded most workers in this field to ignore the data and rely on the output of numerical simulations even though current SN theory may still be missing an important piece of input physics to obtain robust explosions [38].

One explanation for the tension between the KII and IMB data is that the detectors actually observed different spectra due to the Earth matter effect [16,17]. However, our current understanding of flavor-dependent neutrino spectra formation suggests that the flavor-dependent average energies in the antineutrino sector are not very different [18,19]. Moreover, if the observed $\bar{\nu}_e$ had been born as higher-energy $\bar{\nu}_{\mu,e}$ at the source, the tension between theoretically expected and actually observed $\bar{\nu}_e$ energies would be worse.

Loredo and Lamb [14] have tested a large variety of neutrino emission models and, in particular, have included what amounts to a bi-modal spectral shape that contains a lower-energy component attributed to the SN accretion phase and a higher-energy one attributed to the neutron-star cooling phase. They stress that such a spectral form is strongly favored by the data relative to a single-mode thermal spectrum. However, numerical simulations do not predict a bi-modal spectrum because the average energies continuously increase from the accretion to the neutron-star cooling phase and at late times decrease again. Therefore, the spectral shape of the time-integrated flux does not seem to exhibit a bi-modal form but rather is expected to be a broadened quasithermal spectrum.

### Table III

Best-fit values for $E_{\text{tot}}$ in $10^{52}$ erg and $E_0$ in MeV for the indicated fixed choices of $\alpha$. The implied characteristics of the expected detector signals are also shown.

<table>
<thead>
<tr>
<th>Data Set</th>
<th>Best-fit</th>
<th>KII</th>
<th>IMB</th>
<th>IMB</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha = 0$</td>
<td>$E_{\text{tot}}$</td>
<td>$E_0$</td>
<td>$\langle E_{\nu} \rangle$</td>
<td>$N_{\text{events}}$</td>
</tr>
<tr>
<td>KII</td>
<td>17.4</td>
<td>3.7</td>
<td>10.9</td>
<td>15.3</td>
</tr>
<tr>
<td>IMB</td>
<td>23.7</td>
<td>5.0</td>
<td>28.7</td>
<td>18.3</td>
</tr>
<tr>
<td>Joint</td>
<td>10.1</td>
<td>5.4</td>
<td>14.1</td>
<td>19.2</td>
</tr>
<tr>
<td>$\alpha = 2$</td>
<td>$E_{\text{tot}}$</td>
<td>$E_0$</td>
<td>$\langle E_{\nu} \rangle$</td>
<td>$N_{\text{events}}$</td>
</tr>
<tr>
<td>KII</td>
<td>8.9</td>
<td>7.9</td>
<td>11.0</td>
<td>15.3</td>
</tr>
<tr>
<td>IMB</td>
<td>8.2</td>
<td>11.6</td>
<td>20.6</td>
<td>20.1</td>
</tr>
<tr>
<td>Joint</td>
<td>5.9</td>
<td>11.2</td>
<td>14.0</td>
<td>19.5</td>
</tr>
<tr>
<td>$\alpha = 4$</td>
<td>$E_{\text{tot}}$</td>
<td>$E_0$</td>
<td>$\langle E_{\nu} \rangle$</td>
<td>$N_{\text{events}}$</td>
</tr>
<tr>
<td>KII</td>
<td>6.6</td>
<td>10.2</td>
<td>11.0</td>
<td>15.4</td>
</tr>
<tr>
<td>IMB</td>
<td>4.7</td>
<td>15.9</td>
<td>16.3</td>
<td>21.8</td>
</tr>
<tr>
<td>Joint</td>
<td>4.7</td>
<td>14.2</td>
<td>14.1</td>
<td>19.8</td>
</tr>
</tbody>
</table>

FIG. 4. Contours of 95% C.L. in the plane of $E_0$ and $E_{\text{tot}}$ for $\alpha = 0$ (top panel), $\alpha = 2$ (middle), and $\alpha = 4$ (bottom). The dotted lines are for KII and IMB separately whereas the solid line is for the joint data set.
Motivated by this observation we have analyzed the SN 1987A data, assuming a quasithermal spectrum of the general form Eq. (2) that is flexible enough to accommodate a continuum of spectral shapes from narrow (pinched) to broadened (antipinched) spectra relative to a Maxwell-Boltzmann distribution.

Perhaps unsurprisingly in view of the tension between the KII and IMB data, we find that the broadest possible distribution allowed by the chosen prior range of $\alpha$ is preferred. In this way the tension between the data sets is somewhat reduced, but at the same time the average $\bar{\nu}_e$ energies are pulled to lower values, thus exacerbating the tension with the output of representative numerical simulations. (The average neutrino energies found in many different numerical simulations have been collected in Ref. [19].)

Assuming the time-integrated neutrino flux from an SN indeed exhibits a quasithermal spectrum roughly of the form Eq. (2), the implied average $\bar{\nu}_e$ energies and total emitted energy depend sensitively on the chosen prior range for $\alpha$. The data themselves prefer the smallest possible $\alpha$-values, i.e. not a quasithermal spectrum but rather a monotonically falling one. The assumption of a realistic quasithermal spectrum is not simultaneously consistent with typical theoretical expectations for $\langle E_{\nu_e}\rangle$ as well as the separate data sets from IMB and KII. Therefore, it remains unresolved if the SN 1987A data or theoretical simulations give us better benchmarks for the average $\bar{\nu}_e$ energies to be used, for example, in the context of searches for the cosmic diffuse SN neutrino background.

It appears that the question of the true neutrino spectrum from a typical SN can be empirically resolved only by the high-statistics signal from a future galactic SN or by the patient accumulation of data on a neutrino-by-neutrino basis from SNe in nearby galaxies [39].

**ACKNOWLEDGMENTS**

We acknowledge partial support by the Deutsche Forschungsgemeinschaft under Grant No. SFB-375 and by the European Union under the Ilias project, Contract No. RI3-CT-2004-506222. The work of A. M. is supported in part by the Italian “Istituto Nazionale di Fisica Nucleare” (INFN) and by the “Ministero dell’Istruzione, Università e Ricerca” (MIUR) through the “Astroparticle Physics” research project.


