Cosmological Neutrinos

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Abstract

In the framework of the standard cosmological model, large-scale structure observations imply neutrino mass limits in the range \( \sum m_\nu < 0.4–2.0 \text{ eV} \) (95\% CL), depending on the included data sets. Future observations will further improve these sensitivities and may well lead to a positive neutrino-mass detection. The measured neutrino oscillation parameters in conjunction with the big-bang nucleosynthesis constraint on the primordial \( \nu_e \) chemical potential exclude a significant deviation from the standard cosmological neutrino density. Therefore, large-scale structure limits on the hot dark matter fraction indeed translate directly into neutrino mass limits. While neutrinos with sub-eV masses are almost negligible for the cosmic dark matter inventory, they fit nicely with leptogenesis scenarios for creating the baryon asymmetry of the universe.

1. Cosmological Neutrino Mass Limit

In a classic paper, Gershtein and Zeldovich (1966) for the first time used cosmological data to constrain neutrino masses, notably the mass of the muon neutrino that had just been discovered [1]. The cosmic number density of neutrinos plus anti-neutrinos per flavor is \( n_{\nu\bar{\nu}} = \frac{3}{4} n_\gamma \) with \( n_\gamma \) the number density of cosmic microwave photons, and assuming that there is no neutrino chemical potential. With \( T_\gamma = 2.728 \text{ K} \) this translates to \( n_{\nu\bar{\nu}} = 112 \text{ cm}^{-3} \). If neutrinos have masses one finds a cosmic mass fraction

\[
\Omega_\nu h^2 = \sum_{\text{flavors}} \frac{m_\nu}{92.5 \text{ eV}},
\]

where as usual \( h \) is the Hubble constant in units of 100 km s\(^{-1}\) Mpc\(^{-1}\). The requirement that neutrinos do not “overclose” the universe then leads to the approximate limit \( \sum m_\nu < 40 \text{ eV} \).

Later Cowsk and McClelland (1973) speculated that massive neutrinos could actually be the dark matter of the universe [2]. However, neutrinos are not good for this purpose. First, the phase space for neutrinos gravitationally bound to a galaxy is limited [3]. As a consequence, if massive neutrinos are supposed to be the dark matter in galaxies, they must obey a lower mass limit of some 30 eV for typical spirals, and even 100–200 eV for dwarf galaxies. The tritium limit on the \( \nu_e \) mass” of about 2.3 eV (95\% CL) [4, 5] and the very small neutrino mass differences implied by the oscillation experiments show that all neutrino masses are far too small to play a dominant role in galaxies.

The second argument against neutrino dark matter relies on cosmic large-scale structure. The observed structure in the distribution of cosmic matter, and notably in the distribution of galaxies, is thought to arise from the gravitational instability of primordial density fluctuations. The small masses of neutrinos imply that they stay relativistic for a long time after their decoupling (“hot dark matter”), allowing them to stream freely, thereby erasing the primordial density fluctuations on small
scales [6]. While this effect does not preclude neutrino dark matter, it implies a top-down scenario for structure formation where large structures form first, later fragmenting into smaller ones. It was soon realized that the predicted properties of the large-scale matter distribution did not agree with observations and that neutrino dark matter was ruled out [7].

Today it is widely accepted that the universe has critical density and that its matter inventory sports several nontrivial components. Besides some 4% baryonic matter (most of it dark) there are some 24% cold dark matter in an unidentified physical form and some 72% of a negative-pressure component (“dark energy”). And because neutrinos do have mass, they contribute at least 0.1% of the critical density. This fraction is based on a hierarchical mass scenario with \( m_3 = 40 \text{ meV} \), the smallest value consistent with atmospheric neutrino oscillations [16].

An upper limit on the neutrino dark matter fraction can be derived from the measured power spectrum \( P_M(k) \) of the cosmic matter distribution. Neutrino free streaming suppresses the small-scale structure by an approximate amount [8]

\[
\frac{\Delta P_M}{P_M} \approx -8 \frac{\Omega_\nu}{\Omega_M},
\]

where \( \Omega_M \) is the cosmic mass fraction in matter, i.e. excluding the dark energy. The exact limit on the neutrino masses from the absence of any noticeable suppression of small-scale structure depends on the used data for the matter power spectrum and on additional information on global cosmological parameters. Perhaps the most conservative approach is to use only the WMAP data on the cosmic microwave temperature fluctuations, leading to a 95% C.L. upper mass limit of \( \sum m_\nu < 2.0 \text{ eV} \) as shown in Table 1. Including additional data on smaller scales improves the limit. The most restrictive limit of 0.4 eV arises when including the Ly-\( \alpha \) data from the Sloan Digital Sky Survey (Table 1). The most important point is that all of these limits are consistent with each other, the differences being due to the different data sets. Put another way, all high-quality cosmological data are at present consistent with the assumption of a negligibly small neutrino mass.

<table>
<thead>
<tr>
<th>Authors</th>
<th>Ref.</th>
<th>Upper limit ( \sum m_\nu ) [eV]</th>
<th>Used data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ichikawa, Fukugita, Kawasaki</td>
<td>[9]</td>
<td>2.0</td>
<td>WMAP</td>
</tr>
<tr>
<td>Tegmark et al. (SDSS Collab.)</td>
<td>[10]</td>
<td>1.8</td>
<td>WMAP, SDSS</td>
</tr>
<tr>
<td>Spergel et al. (WMAP Collab.)</td>
<td>[11]</td>
<td>0.69</td>
<td>WMAP, CMB, 2dF, HST, Bias</td>
</tr>
<tr>
<td>Barger, Marfatia, Tregre</td>
<td>[12]</td>
<td>0.75</td>
<td>WMAP, CMB, 2dF, SDSS, HST</td>
</tr>
<tr>
<td>Crotty, Lesgourgues, Pastor</td>
<td>[13]</td>
<td>1.0</td>
<td>WMAP, CMB, 2dF, SDSS</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.6</td>
<td>+ HST, SNe Ia</td>
</tr>
<tr>
<td>Hannestad</td>
<td>[14]</td>
<td>0.65</td>
<td>WMAP, SDSS, SNe Ia (gold sample)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Ly-( \alpha ) (Keck sample)</td>
</tr>
<tr>
<td>Seljak et al.</td>
<td>[15]</td>
<td>0.42</td>
<td>WMAP, SDSS, Bias, Ly-( \alpha ) (SDSS sample)</td>
</tr>
</tbody>
</table>

On the other hand, from the oscillation experiments we know that neutrinos do have masses [16]. The largest neutrino mass eigenstate is bounded from below as \( m_\nu > 40 \text{ meV} \) (3\( \sigma \)). Therefore, it is important to ask what it would take to detect non-vanishing neutrino masses in cosmological data. Sensitivity forecasts for different scenarios of future surveys are summarized in Table 2. We conclude
that neutrino masses should eventually show up in cosmological structure data and conversely, $\sum m_\nu$ will become a non-trivial fitting parameter for standard cosmological models. At present ignoring neutrino masses does not affect the parameter estimates of the cosmological concordance model.

### Table 2. Cosmological sensitivity forecasts for detecting non-vanishing neutrino masses.

<table>
<thead>
<tr>
<th>Authors</th>
<th>Ref.</th>
<th>Future surveys</th>
<th>$\sum m_\nu$ [eV] sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hannestad</td>
<td>[17]</td>
<td>Planck &amp; SDSS</td>
<td>0.12</td>
</tr>
<tr>
<td>Lesgourgues, Pastor, Perotto</td>
<td>[18]</td>
<td>Planck &amp; SDSS</td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ideal CMB &amp; 40 x SDSS</td>
<td>0.13</td>
</tr>
<tr>
<td>Abazajian, Dodelson</td>
<td>[19]</td>
<td>4000 deg$^2$ weak lensing survey</td>
<td>0.1</td>
</tr>
<tr>
<td>Kaplinghat, Knox, Song</td>
<td>[20]</td>
<td>CMB lensing</td>
<td>0.15 (Planck)</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>0.044 (CMBpol)</td>
</tr>
<tr>
<td>Wang, Haiman, Hu, Khoury, May</td>
<td>[21]</td>
<td>Weak-lensing selected sample of $&gt; 10^5$ clusters</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Any neutrino mass limit or sensitivity forecast depends on the assumption that the standard cosmological model with its limited number of parameters correctly accounts for our universe. On the other hand, it is possible that the primordial spectrum of density fluctuations is not accounted for by a pure power law or that other subtle deviations from the standard assumptions exist that may compensate for the effect of a neutrino mass or mimic one. The observed universe as a neutrino scales may suffer from systematics that have not yet been fully appreciated. Surely, however, cosmology has begun to probe the sub-eV range of neutrino masses in earnest.

One can also constrain possible other hot dark matter components. Hypothetical axions with masses in the eV-range are a case in point [22, 23] where a mass limit of 1.05 eV was found on the basis of the same cosmological data used by Hannestad [14] to obtain a neutrino mass limit of 0.65 eV (Table 1). This axion limit is complementary to previous limits from SN 1987A [24] and to the future “high-mass” search range of the CERN Axion Solar Telescope (CAST) [25].

The average cosmic neutrino density of a given flavor is $n_{\nu\bar{\nu}} = 112$ cm$^{-3}$. However, neutrinos with masses may no longer be relativistic today and therefore cluster somewhat along with the cold dark matter. Therefore, in our galactic environment the actual neutrino density would be enhanced relative to the cosmic average [26, 27]. Of course, the same applies to hypothetical bosonic hot dark matter contributions such as axions [28].

### 2. How Many Neutrinos in the Universe?

One possible systematic uncertainty of the cosmological mass limits consists of the very number density $n_{\nu\bar{\nu}}$ of neutrinos in our universe. Even though there are exactly three neutrino flavors as indicated by the $Z^0$ decay width, the assumption of thermal equilibrium in the early universe alone does not fix the neutrino abundance. Each flavor is characterized by an unknown chemical potential $\mu_\nu$, or equivalently a degeneracy parameter $\xi_\nu = \mu_\nu/T$, the latter being invariant under cosmic expansion. While the very small baryon-to-photon ratio of about $10^{-9}$ suggests $\xi \ll 1$ for all fermions, for neutrinos this is an assumption and not an established fact.

In the presence of a degeneracy parameter $\xi_\nu$ the thermal number and energy densities of one
species of relativistic neutrinos plus anti-neutrinos are
\[ n_{\nu\overline{\nu}} = T^3_{\nu} \frac{3\zeta_3}{2\pi^2} \left[ 1 + \frac{2\ln(2)}{3\zeta_3} + \frac{\xi^4_{\nu}}{72\zeta_3} + O(\xi^6_{\nu}) \right], \]
\[ \rho_{\nu\overline{\nu}} = T^4_{\nu} \frac{7\pi^2}{120} \left[ 1 + \frac{30}{7} \left( \frac{\xi_{\nu}}{\pi} \right)^2 + 15 \left( \frac{\xi_{\nu}}{\pi} \right)^4 \right]. \]

Therefore, \( n_{\nu\overline{\nu}} \) can only be larger than the standard value. One may think that the cosmological mass limits are thus conservative in that a limit on the hot dark matter fraction would translate into a smaller limit on the neutrino mass. However, the increased radiation content affects the structure-formation limits in a counter-intuitive way. If the number density of neutrinos is larger than standard, the limit on \( \sum m_\nu \) weakens [29]. Therefore, the allowed range of \( \xi_{\nu} \) for the different flavors affects the cosmological mass limits.

Big-bang nucleosynthesis (BBN) is affected by \( \rho_{\nu\overline{\nu}} \) in that a larger neutrino density increases the primordial expansion rate, thereby increasing the neutron-to-proton freeze-out ratio \( n/p \) and thus the cosmic helium abundance. Therefore, the observed helium abundance provides a limit on \( \rho_{\nu\overline{\nu}} \) that corresponds to some fraction of an effective extra neutrino species. In addition, however, an electron neutrino chemical potential modifies \( n/p \propto \exp(-\xi_{\nu_e}) \). Depending on the sign of \( \xi_{\nu_e} \) this effect can increase or decrease the helium abundance and can compensate for the \( \rho_{\nu\overline{\nu}} \) effect of other flavors [30]. If \( \xi_{\nu_e} \) is the only chemical potential, BBN provides the limit [31]
\[ -0.04 < \xi_{\nu_e} < 0.07. \]

Including the compensation effect, the only upper limit on the radiation density comes from precision measurements of the power spectrum of the temperature fluctuations of the cosmic microwave background radiation and from large-scale structure measurements so that the regions \(-0.01 < \xi_{\nu_e} < 0.22 \) and \( |\xi_{\nu_{\mu,\tau}}| < 2.6 \) are allowed [32].

However, the observed neutrino oscillations imply that the individual flavor lepton numbers are not conserved and that in true thermal equilibrium all neutrinos are characterized by one single chemical potential \( \xi_{\nu} \). If flavor equilibrium is achieved before \( n/p \) freeze-out the restrictive BBN limit on \( \xi_{\nu_e} \) applies to all flavors, i.e. \( |\xi_{\nu}| < 0.07 \), implying that the cosmic number density of neutrinos is fixed to within about 1%. In that case the relation between \( \Omega_{\nu} \) and \( m_\nu \) is uniquely given by the standard expression Eq. (1).

The approach to flavor equilibrium in the early universe by neutrino oscillations and collisions was studied by several groups [33, 34, 35, 36]. The detailed treatment is rather complicated and involves a number of subtleties related to the large weak potential caused by the neutrinos themselves as they oscillate. The intriguing phenomenon of synchronized flavor oscillations [37, 38] plays an important and subtle role. The practical bottom line, however, is rather simple. Effective flavor equilibrium before \( n/p \) freeze-out is reliably achieved if the solar oscillation parameters are in the favored LMA region. In the LOW region, the result depends sensitively on the value of the small but unknown third mixing angle \( \Theta_{13} \). In the SMA and VAC regions equilibrium is not achieved. After the KamLAND measurements the solar LMA solution is now singled out as the final answer to the solar neutrino problem [16, 39]. Therefore, if one accepts the above chain of arguments, neutrino chemical potentials no longer constitute a systematic uncertainty for the cosmological mass limit.

The situation is different if one speculates about the existence of additional neutrino states in the form of low-mass right-handed states (sterile neutrinos). They can be thermally excited in
the early universe by oscillations and collisions if they mix with the ordinary active neutrinos—for a recent detailed study see Ref. [40]. A small $\nu_e$ chemical potential, perhaps equally shared among all active and sterile neutrinos, can compensate the BBN expansion-rate effect of the additional energy density in sterile neutrinos. Therefore, in this scenario it is not excluded that the radiation density of the early universe could be larger than is usually assumed and that the cosmological neutrino mass bounds are somewhat relaxed.

3. Neutrinos and the Baryonic Matter

Neutrino masses in the sub-eV range can play an interesting albeit indirect role for creating the baryon asymmetry of the universe (BAU) in the framework of leptogenesis scenarios [41]. The main ingredients are those of the usual see-saw scenario for small neutrino masses. Restricting ourselves to a single family, the relevant parameters are the heavy Majorana mass $M$ of the ordinary neutrino’s right-handed partner and a Yukawa coupling $g_\nu$ between the neutrinos and the Higgs field $\Phi$. The observed neutrino then has a Majorana mass

$$m_\nu = \frac{g_\nu^2 \langle \Phi \rangle^2}{M} \quad (6)$$

that can be very small if $M$ is large, even if the Yukawa coupling $g_\nu$ is comparable to that for other fermions. Here, $\langle \Phi \rangle$ is the vacuum expectation value of the Higgs field which also gives masses to the other fermions.

The heavy Majorana neutrinos will be in thermal equilibrium in the early universe. When the temperature falls below their mass, their density gets exponentially Boltzmann suppressed. However, if at that time they are no longer in thermal equilibrium, their abundance will exceed the equilibrium distribution. The subsequent out-of-equilibrium decays can lead to the net generation of lepton number. CP-violating decays are possible by the usual interference of tree-level with one-loop diagrams with suitably adjusted phases of the various couplings. The generated lepton number excess will be re-processed by standard-model sphaleron effects which respect $B - L$ but violate $B + L$. It is straightforward to generate the observed BAU by this mechanism.

The requirement that the heavy Majorana neutrinos freeze out before they are Boltzmann suppressed implies an upper limit on the combination of parameters $g_\nu^2/M$ that also appears in the see-saw formula for $m_\nu$. The out-of-equilibrium condition thus implies an upper limit on $m_\nu$. Detailed scenarios for generic neutrino mass and mixing schemes have been worked out [42, 43]. For a broad range of models, one finds that thermal leptogenesis works only if all light neutrino masses are below 0.1 eV [44], consistent with neutrino oscillation data.

In summary, neutrino mass and mixing schemes suggested by the atmospheric and solar oscillation data are nicely consistent with plausible leptogenesis scenarios. Of course, it is an open question of how one would go about to verify or falsify leptogenesis as the correct baryogenesis scenario. Measurements of the primordial helium abundance, if taken at face value, suggest a large neutrino asymmetry. If these indications could be substantiated by more reliable and more precise measurements, leptogenesis would be ruled out [31] because the operation of sphaleron effects implies that the baryon and lepton asymmetries are both similar, i.e. of order $10^{-9}$. On the other hand, if neutrinoless double beta decay would be experimentally found, i.e. if the Majorana nature of neutrinos would be reliably established, this would lend great credibility to the leptogenesis scenario. Of course, the claimed evidence for neutrinoless 2$\beta$ decay in the Heidelberg-Moscow Germanium experiment [45] implies a neutrino mass larger than favored by leptogenesis unless the nuclear matrix
element is unexpectedly large. Whatever the final answer to these questions, for now it is exciting that massive neutrinos may have a lot more to do with the baryons than with the dark matter of the universe!

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References


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