Imprint of sterile neutrinos in the cosmic microwave background radiation

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The existence of low-mass sterile neutrinos is suggested by the current status of solar and atmospheric neutrino anomalies and by the Liquid Scintillation Neutrino Defector (LSND) experiment [3]. Taken together, these three bits of evidence are too much of a good thing in that they are incompatible with a three-flavor nucleosynthesis background of sterile neutrinos. The standard big bang neutrino oscillation scenario thus provides nontrivial limits on the masses and mixing parameters that some of these preliminary indications may be unrelated to neutrino oscillations, one intriguing speculation is that there is a fourth low-mass neutrino, $\nu_s$, which mixes with the standard flavors [4]. It would have to be sterile with regard to the electroweak interactions and thus is undetectable in any direct search experiment.

The mixing of $\nu_s$ with standard flavors allows for its thermal production in the early universe, and even though it will typically not attain full equilibrium there will be a cosmic background of sterile neutrinos. The standard big bang nucleosynthesis (BBN) constraint on the cosmic radiation density thus provides nontrivial limits on the masses and mixing angles of a four-neutrino scenario consisting of $\nu_e, \nu_\mu, \nu_\tau$, and $\nu_s$ [5–8]. Likewise, if neutrinos are Dirac particles and thus right-handed components and if they have anomalous magnetic dipole moments, a cosmic abundance of the sterile states can be produced by magnetically induced spin precessions and by electromagnetic spin-flip scatterings [9].

However, the most spectacular cosmological consequence of sterile neutrinos is their impact on the large-scale structure of the universe, and notably on the temperature variations of the cosmic microwave background radiation (CMBR). The anticipated sky maps of the future Microwave Anisotropy Probe (MAP) and PLANCK [10] satellite missions have already received advance praise as the ‘‘cosmic Rosetta stone’’ [11] because of the wealth of cosmological precision information they are expected to reveal [12–15]. In the previous discourse on sterile neutrinos it has been curiously overlooked that a successful deciphering of the CMBR hieroglyphs could well make or break the hypothesis of this elusive particle’s existence. Even if its signature in real CMBR sky maps may not be unambiguously visible, the hypothesis of sterile neutrinos introduces two additional degrees of freedom into the game of cosmological parameter estimation, viz. a hot dark matter component and additional radiation in the form of neutrinos.

II. SENSITIVITY TO RADIATION CONTENT

CMBR sky maps are characterized by their fluctuation spectrum $C_l = \langle a_{lm}^* a_{lm} \rangle$ where $a_{lm}$ are the coefficients of a spherical-harmonic expansion. Figure 1 (solid line) shows $C_l$ for standard cold dark matter (SCDM) with $h=0.5$ for the Hubble constant in units of $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\Omega_{\text{M}}=1$ and $\Omega_{\text{B}}=0.05$ for the matter and baryon content, a Harrison-Zeldovich spectrum of initial density fluctuations, ignoring reionization, and taking $N_{\text{eff}}=3$ for the effective number of thermal neutrino degrees of freedom.

Sterile neutrinos increase the radiation content and thus modify this pattern in a characteristic way illustrated by the dotted line in Fig. 1 which corresponds to $N_{\text{eff}}=4$. While this shift appears small, the lower panel of Fig. 1 shows that for $l\approx 200$ it is large on the scale of the expected measurement precision. It is fundamentally limited by the ‘‘cosmic variance’’ $\Delta C_l/C_l = \sqrt{2l+1}$, i.e. by the fact that at our given location in the universe we can measure only $2l+1$ numbers $a_{lm}$ to obtain the expectation value $\langle a_{lm} a_{lm}^* \rangle$. The actual sensitivity will be worse, but the cosmic variance gives us an optimistic idea of what one may hope to achieve.

The true sensitivity to $\Delta N_{\text{eff}}$ is further limited by our lack of knowledge of several other cosmological parameters. Even then it is safe to assume that we are sensitive to $|\Delta N_{\text{eff}}| \leq 0.3$, and much better with prior knowledge of other parameters [13]. Thus it is clear that the CMBR is a more powerful tool to measure $N_{\text{eff}}$ than the standard big-bang nucleosynthesis (BBN) argument which informs us that $|\Delta N_{\text{eff}}| \leq 1$, where the exact limit adopted by various authors depends on their attitude towards the systematic uncertainties of the primordial light-element abundances [16].

The most optimistic assessment of the $\Delta N_{\text{eff}}$ sensitivity that may be achieved with future CMBR experiments was recently put forth in Ref. [17]. It was claimed that without polarization measurements and without priors of other cosmological parameters one could see $\Delta N_{\text{eff}} \leq 0.4$ if the experi-
FIG. 1. Top: CMBR fluctuation spectrum for SCDM with $h = 0.5$, $\Omega_{\text{m}} = 1$, $\Omega_{\Lambda} = 0.05$, and $N_{\text{eff}} = 3$ (solid line). The dotted line is for $N_{\text{eff}} = 4$, and the dashed line when two of these four neutrinos have equal masses corresponding together to $\Omega_{\text{dm}} = 0.2$ ($\Omega_{\text{CDM}} = 0.75$). Bottom: Relative difference of these nonstandard models to SCDM. The shaded band represents the cosmic variance. (Spectra calculated with the CMBFAST [18] package.)

There are several reasons why these assessments are probably overly optimistic. First, the interpretation of the CMBR signal may be significantly affected by foreground emissions. The treatment in Ref. [17] assumes that the primary error in the data will be due to cosmic variance and neglects possible foreground contamination. This is a problem which can only be treated properly once the new data become available since the nature and magnitude of possible foregrounds are not well known at present (for a discussion see Ref. [19]). Second, the explored cosmological parameter space is limited. There are ‘‘degeneracies’’ between the effect of varying several of the dozen or so standard cosmological parameters which determine the CMBR sky maps. These degeneracies can be broken by other observations, for example the anticipated galaxy correlation functions from the Sloan Digital Sky Survey (SDSS) [20]. In the most recent analysis [15] it was claimed that PLANCK-level CMBR observations with polarization information together with SDSS will achieve only a precision of $\Delta N_{\text{eff}} \approx 0.2$ at the $1\sigma$ level. According to this assessment it will be a struggle to beat the BBN precision of the $N_{\text{eff}}$ determination.

In our following discussion we will take the attitude that a $\Delta N_{\text{eff}}$ of a few 0.1 will be detectable, and that a value as small as 0.01 is not ignorable for the cosmological parameter estimation, even if it may not be identifiable from the CMBR sky maps.

FIG. 2. $\Delta N_{\text{eff}}$ caused by primordial $\nu_\alpha-\nu_\beta$ oscillations with $m_\alpha > m_\beta$. Also shown are the 95% C.L. allowed regions for solar small- and large-angle MSW $\nu_\alpha-\nu_\beta$ oscillations [21] (light shade) and the $3\sigma$ allowed region for atmospheric $\nu_\mu-\nu_\tau$ oscillations [22] (dark shade).

III. MASSLESS NEUTRINOS

As a simple generic case we begin with a four-flavor scenario where the masses are so small that all neutrinos are ultra-relativistic at the epoch of matter-radiation equality ($T_{\text{eq}} = 5.5$ eV $\Omega_{\text{m}}h^2$), i.e. $m_\nu \ll 1$ eV. This implies that the only cosmological effect of $\nu_\alpha$ is its contribution to $N_{\text{eff}}$. Calculations of $N_{\text{eff}}$ from primordial $\nu_\alpha-\nu_\beta$-oscillations as a function of the assumed masses and mixing angles have been performed by many authors [5–7]; we follow the simple method of Ref. [6]. The neutrino ensemble is characterized by a single flavor-polarization vector, i.e. the entire ensemble is treated as having the average momentum ($p$) = 3.15T. As long as there are no resonant oscillations this is sufficiently accurate since neutrinos are kept in kinetic equilibrium until long after they decouple from chemical equilibrium. In the case of resonant transitions the situation is complicated by the fact that different momentum modes pass through the resonance at different temperatures.

Figure 2 shows our results for the equivalent number of extra light neutrinos, $\Delta N_{\text{eff}}$, as a function of the oscillation parameters $\sin^2 2\theta$ and $\delta m^2$ where we have taken $m_\beta > m_\alpha$. Also shown are the 95% C.L. regions for the sterile-neutrino MSW solutions of the solar neutrino problem [21] and the $3\sigma$ favored solution of the atmospheric neutrino anomaly from $\nu_\mu-\nu_\tau$-oscillations [22].

The solar small-angle MSW solution would correspond to a $\Delta N_{\text{eff}}$ at the $10^{-3}$ level, which is undetectable even under the most optimistic assumptions. Likewise, the vacuum so-
lution at $\delta m^2 \approx 10^{-10}$ eV$^2$ has no impact whatsoever on the CMBR.

The large-angle Mikheyev-Smirnov-Wolfenstein (MSW) solution would correspond to $\Delta N_{\text{eff}} = 0.1$, perhaps too small to be clearly visible in the CMBR sky maps. However, it could not be ignored when determining the other cosmological parameters.

The atmospheric neutrino anomaly can be explained by $\nu_{\mu} \leftrightarrow \nu_e$ oscillations with nearly maximum mixing and $\delta m^2 = 10^{-3} - 10^{-2}$ eV$^2$ as indicated by the dark-shaded region in Fig. 2. While the contours where calculated for $\nu_{\tau} \leftrightarrow \nu_e$ oscillations, they roughly also apply to the present case if $m_{\nu_{\tau}} > m_{\nu_e}$. We have checked that independently of the sign of $\delta m^2$ the sterile neutrinos reach almost perfect thermal equilibrium so that a $\nu_{\mu} \leftrightarrow \nu_e$ solution of the atmospheric neutrino anomaly should stick out clearly from the CMBR data. This can be seen in Fig. 2 where the atmospheric solution yields a $\Delta N_{\text{eff}} = 0.8$, even for non-resonant oscillations.

It deserves mention that a sterile species can be thermally excited by other mechanisms than a mass term. For instance, if the neutrino had a Dirac magnetic dipole moment, the right-handed components can be brought into thermal equilibrium by spin-flip interactions with the electromagnetic plasma [9]. Using the CMBR one should therefore be able to constrain the neutrino Dirac dipole moment somewhat tighter than with BBN. Likewise, extra radiation can be produced by exotic neutrino decays of the sort $\nu \rightarrow \nu' \phi$ with $\phi$ a new massless boson such as the Majoron. One of us has already explored the imprint of such scenarios on CMBR sky maps [23].

IV. HOT PLUS COLD DARK MATTER (HCDM)

The LSND experiment indicates a mass difference between $\nu_{\mu}$ and $\nu_{\tau}$ of anywhere between about 0.4 and 3 eV [3]. Taking this result as well as the solar and atmospheric anomalies as serious indications for neutrino oscillations leads us naturally to a four-flavor scenario with two neutrino pairs, each consisting of two nearly mass-degenerate states, and with an eV-range mass separation between the pairs [4]. This would imply that neutrinos play a cosmological role as a hot dark matter (HDM) component and as such correct the problem of overproducing small-scale structure which be-devils SCDM models [24]. The small-scale power spectrum of the cosmic matter-density fluctuations will be measured with unprecedented precision by the Sloan Digital Sky Survey [20]. It was recently shown that these measurements may well be sensitive down to the lower end of LSND-inspired neutrino masses [25].

In addition, there would be an imprint in the CMBR fluctuation spectrum [26]. Neutrinos with eV masses are still relativistic at the epoch of matter-radiation equality so that the HDM component in a hot-cold dark matter (HCDM) scenario initially counts toward the cosmic radiation density, and only later to the matter density. Essentially, by giving mass to the neutrinos we have removed matter from the CDM component when holding $\Omega_d = 1$ fixed so that adding neutrino masses mimics extra radiation at the epoch of matter-radiation equality in a standard flat CDM model. This enhances the first Doppler peak via the early integrated Sachs-Wolfe effect in analogy to extra radiation [19]. Of course, beyond the first peak the modification is more intricate, but the main physical effect at large angular scales can be understood in this way.

In an optimistic interpretation of what PLANCK may achieve, the sensitivity to a HDM component may be as good as $\delta \Omega_{\text{HDM}} = 0.02$ [14]. In a 2$\sigma$CDM picture (two mass-degenerate neutrinos as HDM component) we have $\Omega_{\nu} h^2 = 2 m_{\nu} / 93$ eV, implying an optimistic PLANCK sensitivity to a neutrino mass as low as $m_{\nu} \approx 0.25$ eV if $h = 0.5$.

HCDM scenarios remedy the SCDM problem of overproducing small-scale structure, but there are other possible solutions to this problem. Therefore, the primary motivation for a HDM component of eV-mass neutrinos arises from the LSND measurements which in turn suggest a sterile neutrino if the solar and atmospheric indications are taken seriously as well. (In order to avoid sterile neutrinos, many authors would rather discard the LSND results than any of the other two hints for oscillations; the conflict with the KARMEN limits [27] is getting difficult to ignore.) As a consequence, four-flavor neutrino mass schemes and HCDM scenarios are closely intertwined hypotheses.

For example, if the atmospheric neutrino anomaly is due to $\nu_{\mu} \leftrightarrow \nu_e$ oscillations, we will have approximately $N_{\text{eff}} = 4$, and two of these states will have an eV-range mass. The CMBR imprint of this scenario is illustrated with the dashed curve in Fig. 1 where we have chosen $\Omega_{\nu} = 0.2$. With $\Omega_{\nu} h^2 = 2 m_{\nu} / 93$ eV and taking $h = 0.5$ this implies $m_{\nu} \approx 2.4$ eV, well within the range suggested by LSND. This value for $\Omega_{\nu}$ gives the best fit to observations of the large scale structure, as noted by several authors [24].

The region around the first acoustic peak is seen to be enhanced substantially compared with the massless $N_{\text{eff}} = 4$ scenario. As explained earlier, giving mass to the neutrinos mimics the effect of extra radiation, at least around the first acoustic peak, so that in an $N_{\text{eff}} = 4$ scenario with massive neutrinos the separate effects add to a larger compound imprint.

Other four-flavor scenarios have a less dramatic impact, notably if the sterile state solves the solar neutrino problem with a small mixing angle or a very small mass difference to $\nu_e$. Still, in any of the data-inspired four-flavor schemes one cannot avoid worrying about both, a HDM component and extra radiation.

For any given mass and mixing scheme one can work out $N_{\text{eff}}$ and the HDM component. However, this can be a complicated task when resonant effects become important which, in turn, depend on the unknown primordial lepton-number asymmetry. It has been shown that resonant oscillations can generate a significant $\nu_{\mu} - \bar{\nu}_{\mu}$ asymmetry which affects the primordial helium production through modified $\beta$ reaction rates [7]. Therefore, in four-flavor scenarios, BBN is not always a faithful probe for the radiation content which we express in terms of $N_{\text{eff}}$. But another way, the BBN-quantity $N_{\text{eff}}$ is an indirect measure of the helium yield, while our $N_{\text{eff}}$ is a measure of the radiation content at the epoch of matter-radiation equality. The two notions can be vastly different.
and are separately important. The main point here is that BBN is sensitive to the flavor of neutrinos whereas the CMBR measures only energy density.

V. CONCLUSION

Low-mass sterile neutrinos are a generic possibility, and indeed required if all current empirical indications for neutrino oscillations are correct. This would imply a cosmological hot dark matter component in the form of massive neutrinos, and nonstandard contributions to the radiation density at the epoch of matter-radiation equality. In contrast with previous discussions, both effects would simultaneously occur and would leave their imprint in the large-scale matter distribution as well as in the CMBR temperature sky maps.

In a four-flavor scenario, the neutrino mass- and mixing scheme can be rather complicated, allowing for involved oscillation phenomena in the early universe because of the possibility of resonant effects. It is thus premature to attempt a complete discussion of all possible cases. However, if one takes the current empirical situation with regard to neutrino parameters seriously at all, then nonstandard neutrino properties will have a large impact on the cosmological observables to be extracted from precision CMBR experiments and galaxy surveys. In some scenarios, the sterile-neutrino imprint will stick out very clearly, in others it may not be possible to disentangle it from other effects. The most difficult-to-detect scenario is where atmospheric neutrinos oscillate from $\nu_\mu$ to $\nu_e$ and solar neutrinos from $\nu_e$ to $\nu_\tau$ with the small mixing angle MSW solution or the vacuum solution.

Even if the signature of sterile neutrinos cannot be unambiguously seen in the CMBR sky maps and galaxy surveys, they still affect the interpretation of these cosmological precision observables. Therefore, the current experimental effort to pin down the neutrino mass spectrum and mixing angles is inseparably interwoven with a precision interpretation of the forthcoming CMBR sky maps.

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