LIGHT NEUTRINOS AS COLD DARK MATTER

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In the majoron model of Gelmini and Roncadelli, the neutrino mean free path in the early universe is very small due to the large v-v scattering rate from majoron exchange. Free streaming of neutrinos would be entirely suppressed, because the free streaming scale is now replaced by a very short diffusion scale. If majorons have a mass larger than the lightest neutrino species, these neutrinos would constitute the dark matter of the universe and would form structure according to a cold dark-matter scenario, in spite of their small mass.

The problem of neutrino masses is an unresolved issue which remains a challenge to experimental physics, particle theory, astrophysics, and cosmology. It is well known that neutrinos with masses of a few tens of eV could constitute the dark matter of the universe [1]. Lower values are excluded from phase-space considerations [2] for particles bound to a galaxy – although lower mass neutrinos could contribute to the dark matter on larger than galactic scales – while higher values would yield an unacceptably large mass density of the present-day universe. One experiment concerning the end point of the electron spectrum in tritium $\beta$-decay [3] has yielded evidence for an electron-neutrino mass in the range $20 \text{ eV} < m_{\nu_e} < 45 \text{ eV}$, a result which supports the speculation that neutrinos, indeed, are the dark matter in the universe, although other experiments have only produced upper limits, the most restrictive currently being [4] $m_{\nu_e} < 18 \text{ eV}$.

Neutrino dark matter, however, poses a severe problem concerning the formation of structure from primeval density fluctuations. The main reason is that the mean free path of neutrinos is much larger than the horizon scale $H^{-1}$ after their decoupling at $T \approx 1 \text{ MeV}$ ($H$ is the usual Hubble parameter). Therefore neutrino free streaming erases all primordial density fluctuations on the scale of the horizon and below until these particles become non-relativistic at a time when the temperature is comparable to their mass. This essentially coincides with the epoch of matter-radiation equality, $T_{eq} \approx 10 \text{ eV \cdot } \Omega h_0^2$ where $\Omega$ is the present mass density of the universe in units of the critical density and $h_0$ is the Hubble parameter in units of $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$. The mass contained within the horizon at this time is of order $m_{\nu} \Omega h_0^2 \approx 10^{15} M_{\odot}$ for $m_{\nu} \approx 20 \text{ eV}$, and this large coherence scale results in correspondingly late formation of structure in the universe [5]. Detailed numerical calculations have shown [6] that in this “hot dark-matter” scenario, galaxy formation occurs at too recent an epoch if consistency with the observed galaxy correlations is required. Attention has therefore turned to “cold dark-matter” [7], referring to particles which become non-relativistic at epochs much earlier than $T_{eq}$, including such candidates as axions and photinos.

Therefore cosmology and the tentative result of a finite electron-neutrino mass seem to disagree. Moreover, it is well to recall that the experimental upper limits on $m_{\nu_e}$ and $m_{\nu}$ are well above what is required to close the universe [8]. While one escape from this conundrum is to question the validity [9] of the experimental result [3] it would be prudent to consider other alternatives in view of the possibility that a cosmologically interesting neutrino mass may yet be resurrected by experiment. One possibility is the suggestion that the hot dark-matter scenario – possibly with other ingredients such as cosmic
strings – was not conclusively disproven [10]. Therefore the dismissal of light neutrinos as dark-matter candidates may have been premature.

We presently wish to point out that there is another way to “save” neutrino dark matter: We question the validity of the assumption of neutrino free streaming. The only neutrino cross sections or reaction rates that have been measured involve the interaction with charged leptons or with hadrons, and the impressive agreement [11] with the predictions of the Glashow–Weinberg–Salam model of weak interactions has rendered this the “standard model”. However, no $\nu - \nu$ cross section has ever been reported due to the enormous difficulty associated with such experiments. Hence there remains considerable room for speculations concerning large non-standard $\nu - \nu$ reaction rates.

Although these speculations may at first appear unmotivated, they are supported by the majoron model of Gelmini and Roncadelli [12,13] (GR) which predicts extremely large $\nu - \nu$ cross sections and yet is consistent with all reported experimental data. In fact, a very recent $\beta \beta$-decay experiment [14] provides preliminary evidence for a three-body decay mode of $^{76}$Ge. If interpreted in the framework of the GR model in conjunction with calculated nuclear matrix elements, it yields a dimensionless coupling constant between $v_e$ and majorons of $g_{ee} = (8 \pm 1) \times 10^{-4}$. Typical cross sections between electron neutrinos would then be of order $g_{ee}^2 E_\nu^2$ with $g_{ee} = g_{ee}^{\text{GR}} / 4 \pi$ and $E_\nu$, the neutrino energy.

In the early universe, $E_\nu \approx T$ such that the mean free path is $l \approx \alpha_{ee}^{-2} T^{-1}$. The time available for neutrino streaming can be estimated as $H^{-1}$ where [16] $H \approx T^2 / m_{\text{pl}}$ with $m_{\text{pl}} = 1.22 \times 10^{19}$ GeV. The relevant diffusion scale is then

$$\lambda_{\text{diff.}} \approx \sqrt{H^{-1}} \approx \alpha_{ee}^{-1} T^{-3/2} m_{\text{pl}}^{1/2},$$

(1)

such that the ratio of the diffusion to the free streaming scales, the latter being approximated as $\lambda_{f.s.} \approx H^{-1}$, is

$$r = \lambda_{\text{diff.}} / \lambda_{f.s.} \approx \sqrt{H} \approx \alpha_{ee}^{-1} (T / m_{\text{pl}})^{1/2}.$$  

(2)

Taking the temperature at which neutrinos become non-relativistic a 20 eV and $\alpha_{ee}=10^{-7}$, we find $r \approx 10^{-8}$ at that temperature. The energy density in the universe is approximately $T^4$ such that the energy within the horizon is about $T^4 H^{-3} m_{\text{pl}} T^{-2}$. At $T \approx 20$ eV this is about $10^{13} M_\odot$, corresponding to the previously mentioned coherence scale at $T_{eq}$. In our case this value is reduced by $r^3$ to about $10^{-3} M_\odot$, where neutrinos, in spite of becoming non-relativistic at a very late epoch, would be indistinguishable from cold dark matter.

As it stands, our scenario is not yet consistent. Large scattering cross sections also imply large annihilation rates such that all neutrinos and majorons eventually convert into the lightest species of this tightly coupled system of particles. Since in the minimal GR model, the majoron is a true Nambu–Goldstone boson it is massless and hence all neutrinos would convert into majorons when they become non-relativistic [13]. As has been pointed out by the authors of the GR model, however, it is not unreasonable to assume a small mass $m_M$ for the majoron [15]. If it is somewhat larger than the lightest neutrino species, these neutrinos could still be the dark matter of the universe [15]. The previous cross sections, however, would have to be multiplied by $(T / m_M)^4$, once the temperature falls below $m_M$, due to the propagator of the intermediate majoron. Therefore our result for the ratio $r$ at $T = m_n$ for the lightest neutrino species is $r \approx \alpha_{ee}^{-1} (m_M / m_n)^2 \times (m_n / m_{\text{pl}})^{1/2}$. Our conclusions remain virtually unchanged unless $m_M \gg m_n$.

Large non-standard annihilation rates have been previously invoked [16] to introduce heavy neutrinos with masses in the keV- to GeV-range as (cold) dark matter by circumventing the Lee–Weinberg bound [17] while satisfying the experimental bounds [8] on $m_n$ and $m_{\nu}$. We conclude from these discussions and from ref. [13] that, in the GR model, the annihilation rates are so large that, indeed, only the lightest species can significantly contribute to the dark matter density of the universe.

In the GR model, the Majorana mass terms occur through the coupling of neutrinos to a Higgs field which, at low temperatures, develops a finite vacuum expectation value $v_{GR}$. The majoron is the Nambu–Goldstone mode of this field after spontaneous symmetry breaking. The Majorana mass terms are then given as $m_{ij} = g_{ij} v_{GR}$ where $i,j = e, \mu, \tau$ and where non-zero off-diagonal entries imply flavor mixing of neutrinos. Besides the preliminary positive measurement of $g_{ee}$ there exist several upper
bounds [18] on various entries of the matrix $g_{ij}$ which are all in the range $10^{-2} – 10^{-1}$. For $v_{i}$ only the trivial bound $g_{vi} < 1$ exists. Therefore it is consistent to assume that the majoron field couples to all neutrino species with a strength similar to $g_{ee}$. However, even if typical values for $g_{ij}$ were smaller than the tentative experimental result for $g_{ee}$, and even if this result cannot be confirmed, our scenario is still consistent. Strong deviations from a hot dark-matter scenario will occur for $r < 10^{-4}$ which translates into $g > 10^{-5} – 10^{-6}$ for a “generic” entry in the matrix $g_{ij}$. Hence, over about two orders of magnitude, there is a window for these Yukawa couplings which is allowed by experimental bounds and in which neutrinos would account for a cold or warm dark-matter scenario.

Experimental upper bounds on a two-body $\beta\beta$-decay mode in conjunction with calculated nuclear matrix elements translate into $g_{ee} v_{GR} < 3.2 \text{ eV}$. With $g_{ee} = 8 \times 10^{-4}$ from the reported three-body decay mode [14] this implies $v_{GR} < 4 \text{ keV}$. Furthermore, majorons would couple to electrons with a dimensionless coupling strength $2v_{GR} m_{\nu} / u^{2}$ where $u \approx 250 \text{ GeV}$ is the scale for the spontaneous symmetry breaking of the standard Higgs doublet. From astrophysical bounds on the analogous axion–electron coupling [20], we infer $v_{GR} < 10 – 20 \text{ keV}$. Therefore the transition to the broken phase of the GR Higgs field in the early universe occurs much later than nucleosynthesis. Hence the neutral components of the GR Higgs field consist, during that epoch, of two massless bosons which, after symmetry breaking, split into the (almost) massless majoron and a “higglet” with a mass around $v_{GR}$. These extra degrees of freedom add to the energy density of the universe and hence to the expansion rate with a weight of $8/7$ effective neutrino species [21]. Since the standard picture of nucleosynthesis marginally allows for one light neutrino species [22] beyond the conventional $v_{e}$, $v_{\mu}$, and $v_{\tau}$, the majoron scenario is at least marginally consistent with this bound. There is no room, however, for a fourth generation of light neutrinos.

The tightest constraint on our scenario comes from lower bounds on the age of the universe. Relating the number of neutrinos in the usual way [23] to the number of photons, we find for the mass of the surviving neutrino species

$$m_{\nu} = \frac{\pi}{2\zeta(3)} \frac{H_{0} m_{\nu}^{2}}{T_{0}^{3}} g_{\nu} (T_{\nu} / T_{0})^{3}$$

$$= 70.5 \text{ eV} \cdot h_{0}^{2} (\Omega_{\nu} / g_{\nu}) (T_{\nu} / T_{0})^{3} .$$

(3)

Here $H_{0} = h_{0} \cdot 100 \text{ km s}^{-1} \text{ Mpc}^{-1}$ is the Hubble parameter today, and $T_{0} = 2.7 \text{ K}$ is the temperature of the microwave background radiation. $\Omega_{\nu}$ is the mass density of neutrinos in units of the critical density necessary to close the universe. $g_{\nu}$ is the number of degrees of freedom for the lightest species, in our case of a Majorana neutrino $g_{\nu} = 2$. In the standard picture [23] $(T_{\nu} / T_{0})^{3} = 11/4$. In our case, the lightest neutrino gets heated by the decay of 4 neutrino and 2 boson degrees of freedom, such that $T_{\nu}^{3}$ is increased by a factor 29/7. Hence $(T_{\nu} / T_{0})^{3} = 77/116$ such that $m_{\nu} = 23.4 \text{ eV} \cdot h_{0}^{2}$. Bounds on the age of the universe limit $h_{0}$ for an assumed value for $\Omega_{\nu}$ for this relationship to constitute an upper bound on $m_{\nu}$.

A lower bound is derived from phase space constraints on particles bound to a galaxy. The usual Tremaine–Gunn [2] bound is relaxed by a factor of $2^{-1/4}$ since the assumption of a collisionless neutrino flow does not apply. Hence we find $m_{\nu} > 120 \text{ eV}$.

$$\sigma_{100}^{1/4} R_{\text{kpc}}^{1/2} g_{\nu}^{1/4}$$

where $\sigma_{100}$ is the linear velocity dispersion in units of $100 \text{ km s}^{-1}$ and $R_{\text{kpc}}$ the core radius of the system in units of 1 kpc. For a galactic halo [2] $\sigma_{100} \approx 2$ and $R_{\text{kpc}} \approx 20$ such that $m_{\nu} \gtrsim 19 \text{ eV}$. This bound can be possibly relaxed by more detailed modelling of the galactic dark-matter halo [24]. A similar argument applied to dwarf galaxies [24] implies much larger neutrino masses, with considerable uncertainty, however, because the dark-matter potential well is poorly known. The requirement that neutrinos are the galactic dark-matter translates into $h_{0}^{2} \Omega_{\nu} \gtrsim 0.80$ which leads, with $\Omega_{\nu} \approx 1$, to $H_{0}^{-1} \lesssim 11 \times 10^{9} \text{ yr}$. In the absence of a cosmological constant, this corresponds to an unacceptably young age for the universe. Therefore, we either have to introduce such a term, and there is no strong argument against this conjecture, or else conclude that neutrinos cannot be the dark matter on galactic scales.

A mass $m_{\nu} > 19 \text{ eV}$, however, is afflicted with a further problem. The quoted bound [19] $g_{ee} v_{GR} < 3.2$ is not supported by the case $v_{GR} \gtrsim 100 \text{ keV}$, which does not support this simple estimate, we refer to the remark after eq. (A6) in ref. [21] for the relevant case $v_{GR} \ll 100 \text{ keV}$.
eV indicates that one has to invoke neutrino mixing effects to account for such a large value of the lightest mass eigenstate. Considering only mixing with one other state, we choose $\nu_e$ for which the relevant experimental bounds are least restrictive. For the mixing angle one has experimentally $\sin^2 2\theta < 0.07$ such that $\theta \approx g_{ee}/g_{\ell\ell} < 0.13$. The lightest mass eigenstate would be $m_1 \approx |g_{ee}^2/g_{\ell\ell}|v_{GR}$. Since bounds from $K^+$-decays indicate $|g_{ee}| < 0.013$, we find $m_1 < 1.7 \times 10^{-3} v_{GR}$ such that we need $v_{GR} > 11$ keV if we require $m_1 > 19$ eV, barely consistent with the astrophysical bounds on $v_{GR}$ and in conflict with the experimental bound which is, however, somewhat uncertain.

An interesting way to cure these problems is to introduce Dirac mass terms and thereby introduce right-handed (RH) neutrino states. Then the constraints on the Majorana mass matrix $g_{ij}v_{GR}$ are irrelevant. Furthermore, we now have $g_1 = 4$ for the lightest species, and the galactic phase-space constraint is relaxed to $m_1 > 16$ eV, bringing this mass well within the range allowed by the Zurich tritium experiment [4]. It is quite natural to assume, furthermore, that the Dirac neutrino masses are arranged in a hierarchy similar to that for the charged lepton (Dirac) masses. If the heaviest mass eigenstates exceed about 1 MeV, as allowed by experimental bounds [8], it would vanish before nucleosynthesis, and all possible problems with the bounds on the number of light degrees of freedom [22] are circumvented. The vanishing of this degree of freedom would heat all species, including photons, so that we may entirely neglect it from the subsequent discussion.

In order to evaluate eq. (3) we finally need to estimate the neutrino temperature after all heavy degrees of freedom have vanished. In order to avoid new problems with nucleosynthesis constraints, we require that the RH states be populated only after weak decoupling. Since they participate in all reactions through the Dirac mass, all rates are suppressed by a factor $(m_1/T)^2$ as long as they are relativistic. Hence a typical reaction rate involving RH neutrinos will be of order $(m_1/T)^2$ with $\alpha_{\nu} = g^2/4\pi$ for a “generic” entry of the matrix $g_{ij}$. This exceeds the expansion rate for $T^3 < \alpha_{\nu}^2 m_1^2/m_p$, and $T < 1$ MeV implies $\alpha_{\nu} m_1 < 10^{-2}$ which is easily satisfied with $\alpha_{\nu} = 10^{-7}$ and $m_1$ in the $10-100$ eV range. When the RH states enter equilibrium, entropy is generated because this process out-of-equilibrium. In other words, the energy stored in the LH neutrino-, majoron-, and higglet-degrees of freedom will be re-distributed among these states and the RH degrees of freedom. Hence we have $g_1 T^4 = g_2 T^4_2$ with the index 1 refers to the epoch before and 2 after this process. With our inventory of two neutrino species and two bosons we have $g_1 = 4 + 16/7$, $g_2 = 8 + 16/7$, and $(T_2/T_1)^4 = 11/18$, so that substantial cooling occurs. The entropy generation is characterized by $s_2/s_1 = g_2 T^2_2/g_1 T^2_1 = (g_2/g_1)^{1/4} = (18/11)^{1/4} \approx 1.13$. Later, the Boltzmann suppression of the heavy degrees of freedom heats the lightest neutrino species, a process which takes place in equilibrium with entropy being conserved. Hence $(T_2/T_3)^3 = g_2/g_3 = 18/17$, whence $(T_2/T_1)^3 = (18/7)(11/18)^{3/4}$. Altogether we have to use $(T_1/T_3)^3 = (11/4)(T_1/T_2)^3 = (7/4)(11/18)^{1/4}$ and $g_1 = 4$ to derive from eq. (3) $m_1 = 27$ eV $h_3^2 \Omega_\nu$. With $m_1 > 16$ eV for the lightest species and $\Omega_\nu = 1$ this is $h_\Omega > 0.76$ and $H_{eq}^2 < 13 \times 10^9$ yr, still an uncomfortably low value in the absence of a cosmological constant.

Another interesting constraint arises from the observation of neutrinos [26] from the supernova 1987-A that has recently occurred in the Large Magellanic Cloud. This is the first time that neutrinos from a supernova have been detected. The duration of the measured pulse of a few seconds gives an upper bound on the possible dispersion due to a finite neutrino mass and due to the interaction with any background matter that may be present. In our case this interaction would be quite strong. For neutrino energies in the $10$ MeV range and a coupling strength of $\alpha_{\nu} \lesssim 10^{-6}$, however, the galactic dark matter halo would still be optically thin – see also the discussion in ref. [15]. The real part of the index of refraction would also be too small to yield any significant dispersion effects. Considering the effect of a finite mass, we note that neutrinos with energy $E$ which differ in energy by $\Delta E \ll E$ arrive with a temporal separation of $\Delta t = l(m/E)^2 \Delta E/E$ where $l \approx 50$ kpc is the distance to the supernova. Numerically this is

$$\Delta t = 0.05 \text{s} \cdot (m_1/1 \text{ eV})^2 (10 \text{ MeV}/E)^2 \Delta E/E.$$ 

With $\Delta E/E = 0.2$ this coincides with the result of Bahcall, Dar and Piran [27]. With our preferred value of $m_1 = 16$ eV and $E = 10$ MeV this yields about
Δt = 3 s, in agreement with the measured pulse duration of a few seconds [26].

Our discussion was guided by the assumption that light neutrinos are the dark matter. In the GR model, we were then led to the conclusion that majorons must have a small mass for our scenario to be consistent. This assumption, however, opens the possibility that the majorons which are the lightest species and that they are the dark matter of the universe [15]. It is not clear, however, whether such a majoron scenario could account for cold dark matter because the scattering cross section for majoron–majoron scattering cannot be specified in the absence of a detailed model of the occurrence of their mass term. As long as they are true Nambu–Goldstone bosons there would be no quartic majoron term in the lagrangian and they would scatter mainly through an amplitude involving a neutrino loop. In the presence of a mass term, however, one may expect large direct majoron–majoron couplings such that, indeed, they may act as cold dark matter. This would also imply that significant Bose-condensation effects could take place in a galaxy whence the galactic phase-space constraint on their mass would not apply. A detailed discussion of this scenario is beyond the scope of our present work.

In conclusion we have shown that the majoron model of Gelmini and Roncadelli offers a unique possibility for reconciling neutrinos with a cold dark matter scenario for the formation of structure in the universe. The relevant neutrino mass is the mass of the lightest species. This could conceivably be the electron neutrino with a mass close to the range reported by the Moscow tritium experiment and consistent with the Zurich experiment. Our scenario is constrained by very tight bounds from the phase space of galaxies, the age of the universe (if one insists on a vanishing cosmological constant), and from the dispersion measure of neutrinos from the supernova 1987-A. It is very interesting that these bounds still appear to leave room for our scenario to be consistent. An interesting possibility to be considered in the future is that majorons are the dark matter, a case which may have all the virtues of our scenario without being afflicted with similarly tight constraints.

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