E_8 and F-Theory GUTs

Based on arXiv:1502.03878. Work done with
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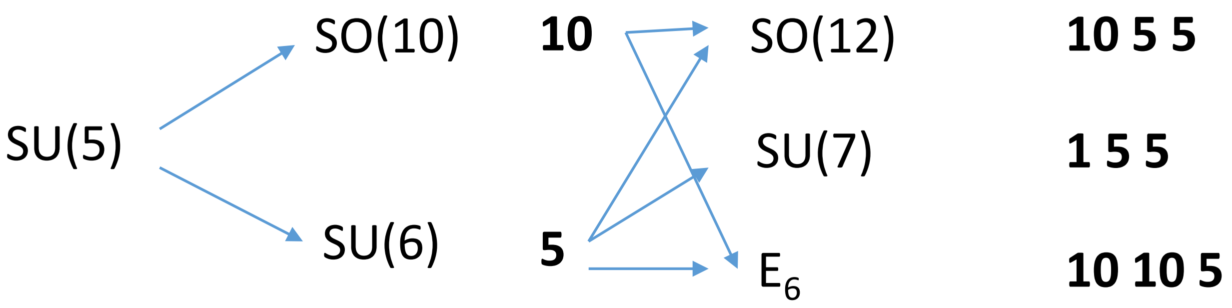
Motivation: Part I

There are few clues or constraints on symmetries beyond the Standard Model – apart from a strong motivation for Grand Unification (minimally SU(5))

Symmetries and their matter representations are some of the most robust and well understood aspects of string theory – perhaps are the likeliest arena for extracting universal predictions/constraints?

F-theory is good framework for addressing such questions: on the line between generality and simplicity

Matter curves and Yukawa couplings are also associated to symmetries in F-theory



- Top quark Yukawa coupling implies the existence of a point of E_6
- Have we extracted the full implications of this?
- Does the unification of Yukawa, matter and gauge fields through symmetries imply a role for exceptional groups in controlling also the matter representations and gauge symmetries?
- A seemingly key question is therefore: given the exceptional structure of the top-quark Yukawa coupling does E_8, as the maximal compact exceptional group, play a role in controlling the matter and gauge symmetries?
- In F-theory this translates to a question of whether the existence of an exceptional co-dimension 3 locus can limit the structure of co-dimension 2 and 1 physics?

Motivation: Part II

Over the last years there has been much work on understanding four-dimensional compactifications of F-theory which exhibit an SU(5) gauge symmetry extended by some Abelian symmetries

The physics data extracted from a typical such fibration is

- A divisor supporting the SU(5) gauge group and a section associated to a U(1)
- Curves on $S_{SU(5)}$ which support massless representations of the symmetry groups present
- An intersection structure of matter curves giving rise to associated Yukawa couplings

There are by now quite a few examples, but it is not clear what the possibilities are? Are they finite? Is there a controlling structure?

Motivation Parts I + II: Is there a connection to E_8?

Higgsing E_8 using its adjoint

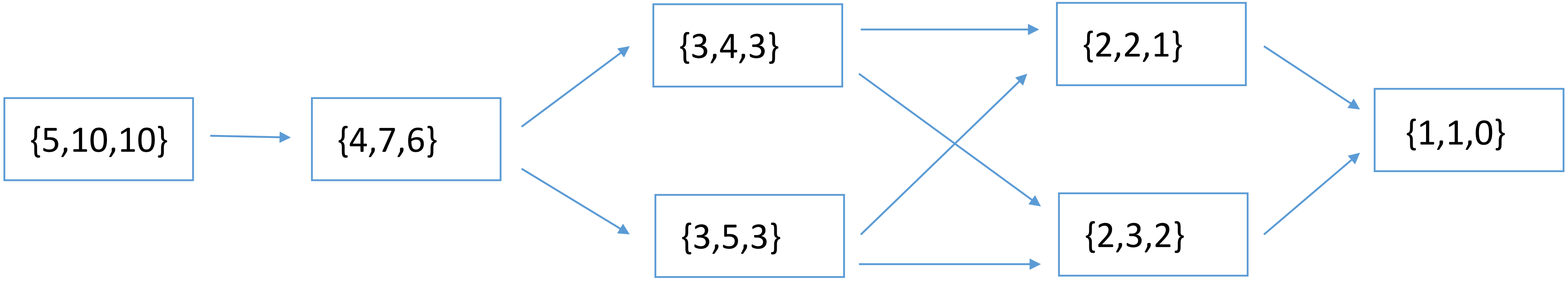
A nice way to parameterise possible $SU(5)xU(1)^n$ theories arising from E_8 is by considering the maximal decomposition to $SU(5)xU(1)^4$ of the adjoint representation

$$E_8 \rightarrow SU(5)_{GUT} \times SU(5)_{\perp} \qquad \mathbf{248} \rightarrow (\mathbf{24},\mathbf{1}) \oplus (\mathbf{1},\mathbf{24}) \oplus (\mathbf{10},\mathbf{5}) \oplus (\bar{\mathbf{5}},\mathbf{10}) \oplus (\bar{\mathbf{10}},\bar{\mathbf{5}}) \oplus (\mathbf{5},\bar{\mathbf{10}})$$

$$U(1)_A = \sum_{i=1}^{5} a_i^A t^i \quad \sum_i a_i = 0 \qquad \mathbf{10}_i : t_i \ , \quad \bar{\mathbf{5}}_{ij} : t_i + t_j \ , \quad \mathbf{1}_{ij} : t_i - t_j \ , \quad t_i t^j = \delta_i^j \, .$$

The possible Higgsing of the U(1)s is determined by vevs for the 10 differently charged GUT singlets

This leads to a Higgsing E_8 tree, where the different charged states are denoted {#10,#5,#1}



Note: we consider only D-flat Higgsing where pairs of singlets get equal vevs

A first guess for a relation to E_8 might be that the F-theory spectra should be embeddable in one of the spectra reached from Higgsing E_8 using its adjoint

This is not possible.

For example, consider a theory with 1 U(1). From E_8 we have 2 possibilities:

4-1 Theory: 10_{-4} , 10_{1} , 5_{-3} , 5_{2} , 1_{5}

3-2 Theory: 10_{-2} , 10_{3} , 5_{-6} , 5_{4} , 5_{-1} , 1_{5}

An example fibration gives:

BGK: 10_{-1} , 5_{-8} , 5_{-3} , 5_{2} , 5_{7} , 1_{5} , 1_{10}

[Braun,Grimm,Keitel '13]

Is there any connection between the spectra of fibrations in the literature and E_8 at all?

An illuminating example

[Mayrhofer, EP, Weigand '12] ([Marsano, Saulina, Schafer-Nameki '09])

There is a set of elliptic fibrations which 'globalise' the Higgsed E_8 theories: Factorised Tate Models

The 3-2 Factorised Tate model can be written in 2 ways. As a Tate form fibration:

$$y^2 = x^3 + a_1xyz + a_2x^2z^2 + a_3yz^3 + a_4xz^4 + a_6z^6$$

$$a_1 = e_2d_3 \, , \quad a_2 = (e_2d_2 + \alpha\delta d_3)\, w \, , \quad a_3 = (\alpha\delta d_2 + \alpha\beta d_3 - e_2\delta\gamma)\, w^2 \, ,$$

$$a_4 = (\alpha\beta d_2 + \beta e_2\gamma - \alpha\delta^2\gamma)\, w^3 \, , \qquad a_6 = \alpha\beta^2\gamma w^5 \, .$$

As a fibration in $P_{[1,1,2]}$:

$$P_T^{[1,1,2]} = z^2 + b_0u^2z + b_1uvz + b_2v^2z + c_0u^4 + c_1u^3v + c_2u^2v^2 + c_3uv^3 = 0$$

$$b_0 = -wd_3\alpha \;\; , \; b_1 = -e_2d_3 \;\; , \; b_2 = \delta \;\; , \; c_0 = w^3\,\alpha\,\gamma \;\; ,$$

$$c_1 = w^2\,(d_2\alpha + e_2\gamma) \;\; , \; c_2 = w\, e_2\, d_2 \;\; , \; c_3 = w\,\beta \, .$$

Both forms share the following matter spectrum:

$$\mathbf{10}^1_{-2} \; : \; w = d_3 = 0 \;\; , \; \mathbf{10}^2_3 : w = e_2 = 0 \; ,$$

$$\mathbf{5}^1_{-6} \; : \; w = \delta = 0, \qquad \mathbf{5}^2_4 \; : \; w = \beta d_3 + d_2 \delta = 0,$$

$$\mathbf{5}^3_{-1} \; : \; w = \alpha^2 c_2 d_2^2 + \alpha^3 \beta d_3^2 + \alpha^3 d_2 d_3 \delta - 2\alpha c_2^2 d_2 \gamma - \alpha^2 c_2 d_3 \delta \gamma + c_2^3 \gamma^2 = 0.$$

$\mathbf{1}^1_5$ $\quad f_1=f_2=0$

$\mathbf{1}^2_{10}$ $\quad \beta = \delta = 0$

Comparing with the 3-2 E_8 model it is based on:

3-2 Theory: 10_{-2} , 10_3 , 5_{-6} , 5_4 , 5_{-1} , 1_5

We find an extra singlet field! Exactly the one needed to make the missing $\mathbf{1}^2_{10}\mathbf{5}^1_{-6}\bar{\mathbf{5}}^2_{-4}$ coupling

This is a general property of many fibrations – one finds for any pair of 5-matter curves there is a singlet which makes such a cubic coupling. Let us call the set of fibrations which exhibit this property “complete networks”.

Higgsing away from E_8

It is possible to deform this fibration in a way which corresponds to Higgsing the non-E_8 singlet

$$P_T^{[1,1,2]} \rightarrow P_T^{[1,1,2]} + c_{4,1} w v^4$$

[Morrison, Taylor '14,...]

And in Tate form:

$$a_4 \;\rightarrow\; a_4 - c_{4,1}\alpha\left(\alpha d_3^2 + 4\gamma w\right) w^3 \,, \qquad a_6 \;\rightarrow\; a_6 + c_{4,1}\left(-\alpha d_2 + e_2\gamma\right)^2 w^5 \,.$$

This leads to a Z_2 gauge theory with spectrum

$$\mathbf{10}_0^1 \,,\; \mathbf{10}_1^2 \,,\; \tilde{\mathbf{5}}_0^1 \,,\; \mathbf{5}_1^3 \,,\; \mathbf{1}_1^1 \,, \qquad \tilde{\mathbf{5}}^1 : \delta\left(\beta d_3 + d_2\delta\right) + e_2 c_{4,1} d_3^2 = w = 0 \,.$$

As expected there is no such theory in the Higgsed E_8 constructions

A proposition for extending E_8

In the decomposition $SU(5)xU(1)^4$ the spectrum exhibits the property that there are not enough singlets to make all possible 1 5 5 couplings

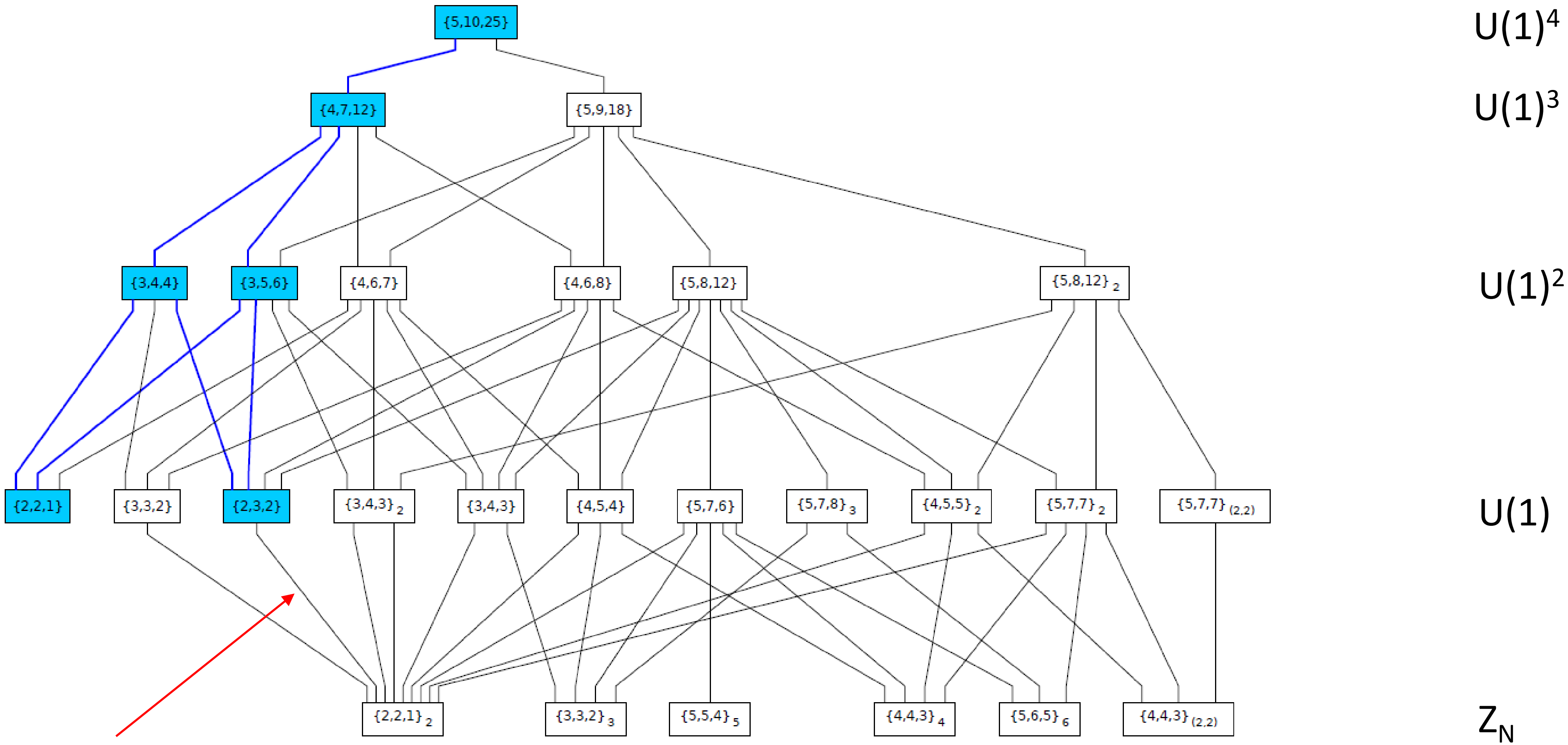
$$\mathbf{10}_i : t_i \ , \quad \bar{\mathbf{5}}_{ij} : t_i + t_j \ , \quad \mathbf{1}_{ij} : t_i - t_j \ ,$$

A natural extension is then to include a new set of 15 GUT singlets with charges such that they allow for a 1 5 5 coupling with all the fields

$$\mathbf{1}_{ijkl} : \ t_i + t_j - t_k - t_l$$

Now Higgsing with these new singlets leads to an extended Higgsing tree

The combinatorics are that 4 choose 25 is 12650, but a computer analysis reveals a surprisingly compact structure



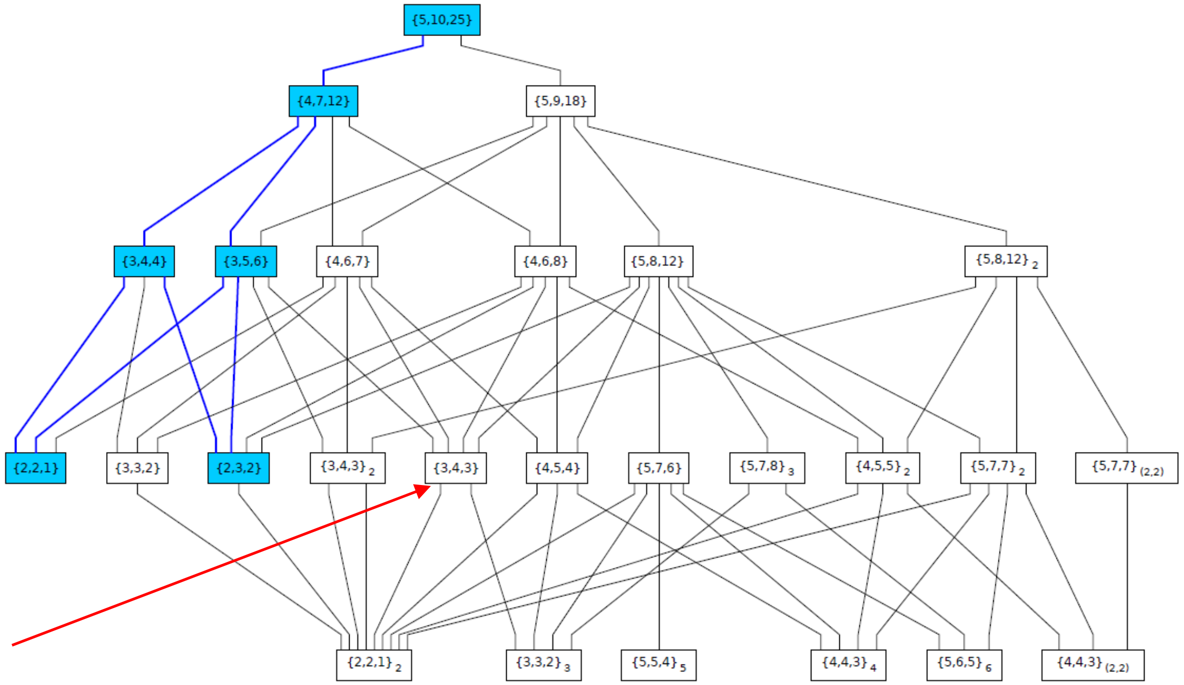
{5,10,25}
{4,7,12}
{5,9,18}
{3,4,4}
{3,5,6}
{4,6,7}
{4,6,8}
{5,8,12}
{5,8,12} 2
{2,2,1}
{3,3,2}
{2,3,2}
{3,4,3} 2
{3,4,3}
{4,5,4}
{5,7,6}
{5,7,8} 3
{4,5,5} 2
{5,7,7} 2
{5,7,7} (2,2)
{2,2,1} 2
{3,3,2} 3
{5,5,4} 5
{4,4,3} 4
{5,6,5} 6
{4,4,3} (2,2)
U(1)4
U(1)3
U(1)2
U(1)
ZN

Embedding known models

Now coming back to our example fibration with no embedding in E_8

BGK: $10_{-1}, 5_{-8}, 5_{-3}, 5_{2}, 5_{7}, 1_{5}, 1_{10}$

We find that it can be embedded in one of the new theories



Overall we looked at 30 fibrations constructed in the literature

[Borchmann, Braun, Cvetic, Garca-Etxebarria, Grassi, Grimm, Kapfer, Keitel, Klevers, Kuntzler, Lawrie, Mayrhofer, EP, Piragua, Sacco, Schafer-Nameki, Till, Weigand]

Find in total that 1/29 and 27/29 of flat fibrations embeddable in E_8 and our extended set respectively

1 fibration not possible to turn off non-flat loci

2 fibrations can not be embedded even in the extended set

Model	spectrum embedded in
No $U(1)$ models	
[24, 25]	$\{2,2,2\}_2$
[25]	$\{2,2,2\}_2$
One $U(1)$ models	
[12]	$\{3,4,2\}$
[19], [22] fiber type $I_5^{(01)}$	$\{3,3,2\}$
[22] fiber type $I_{5,ncnc}^{(01)}$	$\{3,3,2\}$
[19], [22] fiber type $I_5^{(0\vert 1)}$	$\{4,5,4\}$ or $\{\mathbf{2},\mathbf{3},\mathbf{2}\}$
[19], [22] fiber type $I_{5,nc}^{(0\vert 1)}$	$\{\mathbf{2},\mathbf{3},\mathbf{2}\}$
[19], [22] fiber type $I_{5,nc}^{(0\Vert 1)}$	$\{3,4,3\}$
Two $U(1)$'s models	
[11] $4-1$ split	$\{\mathbf{2},\mathbf{2},\mathbf{1}\}$
[11] $3-2$ split	$\{\mathbf{2},\mathbf{3},\mathbf{2}\}$
Top 1	$\{\mathbf{3},\mathbf{5},\mathbf{6}\}$
Top 2	$\{5,8,12\}$
Top 3	$\{4,6,7\}$
Top 4	$\{4,6,8\}$
[26]	$\{5,8,12\}$
$I_5^{s(0\vert 1\Vert 2)}$ $(2,2,2,0,0,0,0,0)$	$\{\mathbf{3},\mathbf{4},\mathbf{4}\}$, $\{4,6,7\}$, $\{5,8,12\}$ *
$I_5^{s(0\vert 1\vert 2)}$ $(2,1,1,1,0,0,1,0)$	$\{\mathbf{3},\mathbf{5},\mathbf{6}\}$
$I_5^{s(0\vert 1\Vert 2)}$ $(2,1,1,1,0,0,1,0)$	$\{5,8,12\}$
$I_5^{s(1\vert 0\vert 2)}$ $(3,2,1,1,0,0,0,0)$	$\{5,8,12\}$
$I_5^{s(01\vert 2)}$ $(3,2,1,1,0,0,0,0)$	$\{4,6,8\}$
$I_5^{s(0\vert 12)}$ $(4,2,0,2,0,0,0,0)$	Not embeddable
$I_5^{s(012)}$ $(5,2,0,2,0,0,0,0)$	Not embeddable
$I_5^{s(01\Vert 2)}$ $(2,2,2,0,0,0,0,0)$	$\{4,6,7\}$
$I_5^{s(0\vert 1\Vert 2)}$ $(2,1,1,1,0,0,0,0)$	$\{\mathbf{3},\mathbf{5},\mathbf{6}\}$ *
$I_5^{s(01\Vert 2)}$ $(2,1,1,1,0,0,0,0)$	$\{4,6,7\}$
$I_5^{s(1\vert 0\vert 2)}$ $(2,1,1,1,0,0,0,0)$	$\{5,8,12\}$
$I_5^{s(0\vert 2\Vert 1)}$ $(1,1,1,1,0,0,1,0)$	$\{5,8,12\}$
$I_5^{s(0\vert 1\Vert 2)}$ $(1,1,1,0,0,0,0,0)$	No consistent way to turn off non-flat points.
[15] 2 Fibrations	Any of the 2 $U(1)$ models

The 2 non-embeddable models were the only ones with the property that they did not form complete networks

[Lawrie, Sacco '14]

$I_5^{s(1\|02)}$	$(4,2,0,2,0,0,0,0)$ $[-,-,\sigma_3\sigma_4,-,\sigma_2\sigma_4+\sigma_3\sigma_5,\sigma_1\sigma_3,\sigma_2\sigma_5,\sigma_1\sigma_2]$	σ_1 σ_2 σ_3 σ_4 σ_5 $\sigma_2\sigma_4-\sigma_3\sigma_5$ $(B.9)$ $(B.10)$	$\mathbf{5}_{0,6}\oplus\overline{\mathbf{5}}_{0,-6}$ $\mathbf{5}_{1,6}\oplus\overline{\mathbf{5}}_{-1,-6}$ $\mathbf{5}_{-1,1}\oplus\overline{\mathbf{5}}_{1,-1}$ $\mathbf{5}_{1,1}\oplus\overline{\mathbf{5}}_{-1,-1}$ $\mathbf{5}_{-1,-4}\oplus\overline{\mathbf{5}}_{1,4}$ $\mathbf{10}_{0,2}\oplus\overline{\mathbf{10}}_{0,-2}$ $\mathbf{5}_{0,-4}\oplus\overline{\mathbf{5}}_{0,4}$ $\mathbf{5}_{0,1}\oplus\overline{\mathbf{5}}_{0,-1}$

The 5-curves σ_1 and σ_2 have no 1 5 5 coupling

So all 27 complete networks were embeddable in our extended set

Less generic configurations need further understanding…

SO(10) Fibrations

It turns out that the lack of singlets in the adjoint of E_8 to form 1 5 5 coupling occurs only for SU(5) as a GUT group

For SO(10) we have a singlet for each pair of 10 representations

$$\mathbf{248} \rightarrow (\mathbf{45}, \mathbf{1}) \oplus (\mathbf{16}, \mathbf{4}) \oplus (\overline{\mathbf{16}}, \overline{\mathbf{4}}) \oplus (\mathbf{10}, \mathbf{6}) \oplus (\mathbf{1}, \mathbf{15})$$

We find, consistently, that 10 SO(10) top models are all embeddable in E_8 upon restriction of flatness

Model	**10**-matter	**16**-matter	**16 16 10** coupling	Non-flat loci
		$SO(10) \times U(1)$ models		
Top 1	b_2 , $c_{1,2}$	$c_{2,1}$	$c_{2,1} \cap b_2$	$c_{2,1} \cap c_{1,2}$
Top 2	$b_{0,2}$, c_3	$c_{2,1}$	$c_{2,1} \cap b_{0,2}$	$c_{2,1} \cap c_3$
Top 3	$b_2 c_{1,2} - b_{0,1} c_{3,1}$	$b_{0,1}$, b_2	$b_{0,1} \cap b_2$, $b_{0,1} \cap c_{1,2}$	$c_{3,1} \cap b_2$
Top 4	$c_{3,1}$	b_2	-	$c_{3,1} \cap b_2$, $b_{0,1}$
Top 5	$c_{1,2}$	$b_{0,1}$	$c_{1,2} \cap b_{0,1}$	b_2
		$SO(10) \times U(1) \times U(1)$ models		
Top 1	b_0 , $d_{1,1}$	d_0	$d_{1,1} \cap d_0$	$d_0 \cap b_0$, $c_{1,1}$
Top 2	c_2 , $b_{2,2} d_0 - c_{1,1} d_{2,1}$	d_0 , $c_{1,1}$	$d_0 \cap c_2$, $d_0 \cap c_{1,1}$	$d_0 \cap d_{2,1}$, $c_{1,1} \cap c_2$, $c_{1,1} \cap b_{2,2}$
Top 3	$b_{2,2}$, c_2	$c_{1,1}$	-	$c_{1,1} \cap b_{2,2}$, $c_{1,1} \cap c_2$, d_0
Top 4	c_2 , $d_{2,1}$	d_0	$c_2 \cap d_0$	$d_0 \cap d_{2,1}$, $c_{1,1}$
Top 5	c_2 , $d_{0,1}$	b_0	-	$b_0 \cap c_2$, $b_0 \cap d_{0,1}$, $b_{2,1}$

[See also Kuntzler, Schafer-Nameki ‘14]

Heterotic Duality – Bundle Data

Since perturbative Heterotic models are based on E_8 it is interesting to think about the Heterotic duals of the beyond-E_8 models

$$y^2 - x^3 = fx + g\,, \qquad\qquad f = \sum_i f_i w^i\,, \quad g = \sum_i g_i w^i\,.$$

Beyond E_8 in bundle data? Find that in Weierstrass form $f_0..f_3$, $g_0...g_5$ encode more data than an SU(5) spectral cover

Concretely starting from a Tate fibration an mapping to Weierstrass one finds

$$y^2 = x^3 + a_1 xyz + a_2 x^2 z^2 + a_3 yz^3 + a_4 xz^4 + a_6 z^6$$

$$\begin{aligned}
f_0 &= -1/48 b_5^4\,, \quad f_1 = -1/12 b_5^2 b_4\,, \quad f_2 = -1/12(b_4^2 - 6b_5 b_3)\,, \quad f_3 = 1/24 b_2\,, \\
g_0 &= 1/864 b_5^6\,, \quad g_1 = 1/144 b_5^4 b_4\,, \quad g_2 = 1/72 b_5^2(b_4^2 - 3b_5 b_3)\,, \\
g_3 &= -1/864(3b_2 b_5^2 - 8b_4^3 + 72 b_5 b_4 b_3)\,, \quad g_4 = 1/144(-b_2 b_4 + 36 b_3^2) + \Delta g_4\,, \\
g_5 &= 1/288 b_0\,,
\end{aligned}$$

Heterotic Duality - Bundle

$$b_i \sim a_{6-i,5-i}$$

$$\begin{aligned}\Delta g_4 \;=\;& 1/576 b_5^2 \left(a_{1,1}^4 + 8a_{1,1}^2 a_{2,2} + 16a_{2,2}^2 - 24a_{1,1}a_{3,3} - 48a_{4,4} + 2a_{1,1}^2 a_{1,2}b_5 + 8a_{1,2}a_{2,2}b_5 \right.\\ & -24a_{3,4}b_5 + 2a_{1,2}^2 b_5^2 + 4a_{1,1}a_{1,3}b_5^2 + 8a_{2,4}b_5^2 + 4a_{1,4}b_5^3 + 8a_{1,1}a_{1,2}b_4 + \\ & \left. 16a_{2,3}b_4 + 8a_{1,3}b_5 b_4 - 24a_{1,2}b_3\right) \;. \end{aligned} \quad (3.5)$$

The sub-leading terms in w are in Tate form are induced by Higgsing away form E_8

$$a_4 \;\rightarrow\; a_4 - c_{4,1}\alpha\left(\alpha d_3^2 + 4\gamma w\right) w^3 \;,$$

For SO(10) or higher the degrees of freedom on each side of the duality are equal

A natural possibility is that Δg_4 relates to the degrees of freedom of Higgsing beyond E_8

Heterotic Duality - Geometry

Non-perturbative gauge symmetries outside of E_8 can arise in the Heterotic string when the bundle degenerates over a singularity

We studied the relation of the singlets extending E_8 with singularities in the Heterotic dual for the 4 top models in [Borchmann, Mayrhofer, EP, Weigand '13]

Find that the heterotic duals have singularities, and that restricting the fibration to turn off the singularities truncates the spectrum so that it is embeddable in E_8

However for the 3-2 factorised Tate model we find no singularity associated to the non-E_8 singlet.

Summary

- Introduced an extension to E_8 which results in a new tree of $SU(5)xU(1)^n$ theories

- All 27 flat (complete network) fibrations in the literature could be embedded into this extended set

- 2 non-complete networks did not have an embedding – a further extended notion of E_8?

- Some comments on Heterotic duality: non-E_8 singlets possibly associated to additional degrees of freedom on the F-theory side of the bundle data, also possibly associated to singular loci on the Heterotic geometry

- Only a first step towards a possible programme of research to extract the full implications of the existence of a Yukawa E_6 point

Thanks

What do we know E_8 is not:

- E_8 is not a maximal gauge symmetry group in String Theory

$$E_8^{2561} \times F_4^{7576} \times G_2^{20168} \times SU(2)^{30200}$$

[Candelas, Perevalov, Rajesh '97]

But in considering a GUT all the charged matter comes from one divisor

- Even on a single E_8 divisor it is still possible to enhance the symmetry further

Heterotic Small Instantons ⟷ M-theory M5-M9(E_8) Collisions ⟷ F-theory non-minimal singularities

But in string theory this seems to always be accompanied by an infinite number of massless states: tensionless strings and their excitations (4D still not fully understood?)