

Introduction to Gauge/Gravity Duality

Examples IV

To hand in Thursday 19th November in the examples class

I. DBI action for a Dp-brane

The Dirac–Born–Infeld (DBI) action for a Dp-brane reads

$$S_{Dp} = -\tau_p \int d^{p+1}\zeta e^{-\Phi} \sqrt{-\det(\mathcal{P}[g]_{ab} + 2\pi\alpha' F_{ab})},$$

where Φ is the dilaton, $g_{\mu\nu}$ the metric of the curved spacetime and \mathcal{P} the Pullback on the worldvolume (given by coordinates ζ^a). Moreover, F_{ab} is the $U(1)$ field strength tensor. \det denotes the determinant of the matrix $\mathcal{P}[g]_{ab} + 2\pi\alpha' F_{ab}$.

a) Simplify the action to a flat target spacetime with a vanishing dilaton. Furthermore, the Dp-brane is aligned along the coordinate axis and the embedding functions into the target spacetime (given by coordinates X^m) vanish, i.e. $X^a = \zeta^a$ for $a = 0, \dots, p$ and $X^m = 0$ for $m = p+1, \dots, 9$. (1 points)

b) Show that we can expand $\det(\mathbb{1} + \epsilon M)$ for small ϵ in the form

$$\det(\mathbb{1} + \epsilon M) = 1 + \frac{1}{2} \epsilon \operatorname{tr} M + \epsilon^2 \left(\frac{1}{8} (\operatorname{tr} M)^2 - \frac{1}{4} \operatorname{tr} (M^2) \right) + \mathcal{O}(\epsilon^3)$$

How does the expansion read for antisymmetric M .

Hint: $\det(A) = \exp(\operatorname{Tr} \ln A)$ and expand the right side of the equation. (2 points)

c) Expand the simplified DBI action of exercise a) using the results of b) up to the first non-trivial order of the field strength tensor F . (2 points)

II. Near Horizon limit of M2-branes

Let us consider the near horizon limit of M2-branes. The supergravity solution of M2-branes reads

$$\begin{aligned} ds^2 &= H(r)^{-2/3} (-dt^2 + dx^2 + dy^2) + H(r)^{1/3} (dr^2 + r^2 d\Omega_7^2), \\ F_{(4)} &= dt \wedge dx \wedge dy \wedge dH^{-1}, \end{aligned}$$

where $H(r)$ is given by

$$H(r) = 1 + \frac{L^6}{r^6}, \quad \text{where } L^6 = 32\pi^2 N l_p^6.$$

a) Take the near horizon limit $r \rightarrow 0$ and calculate the metric and the four-form $F_{(4)}$ in this limit. (3 points)

b) Use the following coordinate transformation $z = \frac{L^3}{2r^2}$ and compute the metric as well as the four-form $F_{(4)}$ in the coordinates (z, t, x, y, Ω_7) . (2 points)