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Introduction to Gauge/Gravity Duality

Examples III

To hand in Thursday 12th November in the examples class

I. **Geodesics in global AdS**

We consider *global AdS* given by the coordinates (ρ, τ, Ω_i) (see also examples II) and the metric

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = L^2 \left(-\cosh^2 \rho \; d\tau^2 + d\rho^2 + \sinh^2 \rho \; d\Omega_i^2 \right) .$$

a) Find the trajectory $\tau(\rho)$ of a massless, radially-directed geodesics, starting from $\rho = \rho_0$ with proper time $\tau(\rho_0)$. What is the coordinate time for a geodesics to go from ρ_0 to the boundary and come back? What is the proper time measured by a stationary observer's clock at ρ_0 for this trajectory? Comment on this!

(2 points)

b) What is the behaviour of a massive geodesics in the radial direction of the AdS? Show that a massive geodesic never reaches the conformal boundary of AdS. Hints:

(i) The norm of the velocity $u^\mu = \frac{dx^\mu}{dT}$ is always -1, where T is the proper time along the worldline.

(ii) We know that $\cosh^2 \rho \; \frac{d\tau}{dT} = C = \text{const.}$ (Why?)

(3 points)

II. **Curvature of AdS and Cosmological constant**

Let us consider AdS_{d+1} in the *Poincare patch* given by the coordinates $(z, t, \vec{x})$ and the metric

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \frac{L^2}{z^2} \left(dz^2 - dt^2 + d\vec{x}^2 \right) ,$$

where $\vec{x}$ are the $d-1$ spatial dimensions.

a) Calculate $g^{\mu\nu}$, the Levi Civita connection $\Gamma^\mu_{\nu\rho}$, the Riemann tensor $R^\mu{}_{\nu\rho\sigma}$ as well as the Ricci tensor $R_{\mu\nu}$ and the Ricci scalar R.

(3 points)

b) Show that AdS_{d+1} solves the vacuum Einstein field equations $G_{\mu\nu} + \Lambda g_{\mu\nu} = 0$, where Λ is the cosmological constant and $G_{\mu\nu} = R_{\mu\nu} - \frac{R}{2} g_{\mu\nu}$ is the Einstein tensor. Determine the value of the cosmological constant Λ!

(2 points)