

## Introduction to Gauge/Gravity Duality

### Examples VIII

To hand in Thursday 16th December in the examples class

#### I. Essay: c-Theorem

Write an essay about the c-Theorem. In particular, state the c-Theorem on the field theory side and sketch briefly the calculation on the dual gravity side. Why is the c-Theorem an important check for AdS/CFT?

Relevant literature is found in

a) c-Theorem and AdS/CFT:

D. Z. Freedman, S. S. Gubser, K. Pilch and N. P. Warner, “Renormalization group flows from holography supersymmetry and a c-theorem,” *Adv. Theor. Math. Phys.* **3** (1999) 363 [arXiv:hep-th/9904017]. Server: <http://arxiv.org/abs/hep-th/9904017>.

b) c-theorem in quantum field theory:

John Cardy: Les Houches Lecture Notes (1988): Conformal Invariance and Statistical Mechanics. May be obtained from John Cardy’s homepage:

<http://www-thphys.physics.ox.ac.uk/people/JohnCardy/>

The c-Theorem is discussed on page 51.

(5 points)

#### II. Fefferman-Graham expansion

Consider the five-dimensional metric

$$ds^2 = L^2 \left( \frac{d\rho^2}{4\rho^2} + \frac{1}{\rho} g_{ij}(x, \rho) dx^i dx^j \right).$$

According to the Fefferman-Graham-Theorem  $g_{ij}(x, \rho)$  can be expanded in the form

$$g_{ij}(x, \rho) = \bar{g}_{ij}(x) + \rho g_{(2)ij}(x) + \rho^2 g_{(4)ij}(x) + \rho^2 \ln \rho h_{(4)ij}(x) + \dots,$$

if the five-dimensional Einstein equations  $R_{\mu\nu} = -4 g_{\mu\nu}$  are satisfied. The functions  $g_{(l)ij}(x)$  and  $h_{(l)ij}(x)$  can be expressed in terms of local functions of  $\bar{g}_{ij}(x)$  and curvature tensors of  $\bar{g}_{ij}(x)$ .

Calculate  $g_{(2)ij}$  by using the Einstein equations!

(5 points)