

Introduction to Gauge/Gravity Duality

Examples V

To hand in Thursday 25th November in the examples class

I. Near Horizon limit of M2-branes

Let us consider the near horizon limit of M2-branes. The supergravity solution of M2-branes reads

$$\begin{aligned} ds^2 &= H(r)^{-2/3} (-dt^2 + dx^2 + dy^2) + H(r)^{1/3} (dr^2 + r^2 d\Omega_7) , \\ F_{(4)} &= dt \wedge dx \wedge dy \wedge dH^{-1} , \end{aligned}$$

where $H(r)$ is given by

$$H(r) = 1 + \frac{L^6}{r^6} , \quad \text{where } L^6 = 32\pi^2 N l_p^6 .$$

- a) Take the near horizon limit $r \rightarrow 0$ and calculate the metric and the four-form $F_{(4)}$ in this limit. (3 points)
- b) Use the following coordinate transformation $z = \frac{L^3}{2r^2}$ and compute the metric as well as the four-form $F_{(4)}$ in the coordinates (z, t, x, y, Ω_7) . (2 points)

II. AdS_5/CFT_4 duality

- a) State the precise AdS_5/CFT_4 duality in the strongest form, i.e. for general ranks N of the gauge group and for arbitrary 't Hooft coupling constants λ ! What is the strong and weak form of the duality? Which limits are taken on both sides of the duality?

Hint: Helpful relations: $g_{YM}^2 = 4\pi g_s$, $\lambda = Ng_{YM}^2$ and $R^4 = 4\pi g_s N \alpha'^2$. (3 points)

- b) What is the operator-field map? What are normalizable and non-normalizable modes and what is their meaning on the field theory side? (2 points)