Priv.-Doz. Dr. Johanna Erdmenger

Introduction to Gauge/Gravity Duality

Examples I

To hand in Thursday 28th October in the examples class

I. **Large N gauge theory**

For the Lagrangian introduced in the lecture (ie. for a scalar field in the adjoint representation of $SU(N)$), draw the double-line version of the vacuum graphs

For each of these graphs, determine the genus and the power of N they correspond to.

(3 points)

II. **Conformal Symmetry**

a) Show that a special conformal transformation may be written as a combination of an inversion, a translation and another inversion.

(2 points)

b) Consider a curved space with a metric of Euclidean signature. An infinitesimal *Weyl transformation* of the metric is given by

$$\delta g^{\mu\nu}(x) = 2\sigma(x) g^{\mu\nu}(x)\,, \tag{1}$$

with a scalar function $\sigma(x)$. Show that the conformal Killing equation may be obtained by combining an infinitesimal diffeomorphism with an infinitesimal Weyl transformation, and subsequently reducing to flat space.

(5 points)

c) Show that the scalar two-point function on flat Euclidean d-dimensional space,

$$\langle \phi(x)\phi(y)\rangle \;=\; \frac{C}{(x-y)^{2\Delta}} \tag{2}$$

is covariant under conformal transformations.

(3 points)

Special question:

d) Graphical representation of special conformal transformations. Write a computer programme which for a given vector $b_\mu \in \mathbb{R}^2$ generates a graphical representation of the special conformal transformation

$$x'_\mu = \frac{x_\mu + b_\mu x^2}{\Omega^b(x)} \,, \qquad \Omega^b(x) = 1 + 2(b \cdot x) + b^2 x^2 \,, \tag{3}$$

of a square lattice.

(3 additional points)