

## Introduction to Gauge/Gravity Duality

### Examples X

To hand in Thursday 21st January in the examples class

#### I. Thermodynamics of the AdS Schwarzschild black hole

Consider the following metric of a  $(d+1)$  dimensional asymptotically AdS Schwarzschild black hole

$$ds^2 = \frac{L^2}{r^2} \left( f(r) d\tau^2 + \frac{dr^2}{f(r)} + dx^i dx_i \right),$$

with  $f(r) = 1 - \left( \frac{r}{r_H} \right)^d$ .

a) Determine the Ricci tensor  $R_{\mu\nu}$  and the Ricci scalar  $R$  (It is not necessary to do a calculation, a good argument is enough)!

(2 points)

b)\* Consider the (euclidean) action

$$S_E = -\frac{1}{2\kappa^2} \int d^{d+1}x \sqrt{g} (R - 2\Lambda) + \frac{1}{2\kappa^2} \int_{r \rightarrow 0} d^d x \sqrt{\gamma} \left( -2K + \frac{2(d-1)}{L} \right),$$

where  $\Lambda = -d(d-1)/(2L^2)$  is the cosmological constant,  $\gamma_{\mu\nu}$  is the induced metric on the boundary  $r \rightarrow 0$ ,  $n^\mu$  is an outward pointing normal vector to the boundary and  $K = \gamma^{\mu\nu} \nabla_\mu n_\nu$  is the trace of the extrinsic curvature.

Compute the on-shell action by plugging the metric of the AdS Schwarzschild black hole into the euclidean action  $S_E$ .

[Result: the euclidean on-shell reads

$$S_E = -\frac{(4\pi)^d L^{d-1}}{2\kappa^2 d^d} V_{d-1} T^{d-1},$$

where  $V_{d-1}$  is the (spatial) volume of the corresponding field theory.] (4 points)

c) Compute the partition function

$$Z = \exp(-S_E)$$

as well as the free energy  $F$  and the entropy  $S$

$$F = -T \log Z, \quad S = -\frac{\partial F}{\partial T}.$$

(4 points)

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\*This is a very tedious exercise. Note, however, that the result is given at the end of the exercise b). Therefore you can skip exercise b) and continue with exercise c).