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## Quantum Fields in Curved Spacetime

### Examples IX

To hand in Monday 15th December in the examples class

#### 1. Schwarzschild Black Hole

Consider a particle (not necessarily on a geodesic) that has fallen inside the event horizon,  $r < 2GM$ . Use the ordinary Schwarzschild coordinates  $(t, r, \theta, \phi)$ . Show that the radial coordinate must decrease at a minimum rate given by

$$\left| \frac{dr}{d\tau} \right| \geq \sqrt{\frac{2GM}{r} - 1}. \quad (1)$$

Calculate the maximum lifetime for a particle along a trajectory from  $r = 2GM$  to  $r = 0$ . Express this in seconds for a black hole with mass measured in solar masses. Show that this maximum proper time is achieved by falling freely with  $E \rightarrow 0$ .

(5 pts)

#### 2. Hawking radiation

a) Estimate the typical wavelength of photons radiated by a black hole of mass  $M$  and compare it with the size of the Schwarzschild radius.

b) The temperature of a sufficiently small black hole can be high enough to produce protons as component of the Hawking radiation. Estimate the required mass  $M$  of such black holes and compare their Schwarzschild radius with the size of the proton (ie. its Compton length).

(5 pts)