Priv.-Doz. Dr. Johanna Erdmenger
Dr. Sergei Winitzki

Quantum Fields in Curved Spacetime

Examples IX

To hand in Monday 15th December in the examples class

1. **Schwarzschild Black Hole**

Consider a particle (not necessarily on a geodesic) that has fallen inside the event horizon, $r < 2GM$. Use the ordinary Schwarzschild coordinates (t, r, θ, ϕ). Show that the radial coordinate must decrease at a minimum rate given by

$$\left|\frac{dr}{d\tau}\right| \geq \sqrt{\frac{2GM}{r} - 1}\,. \tag{1}$$

Calculate the maximum lifetime for a particle along a trajectory from $r = 2GM$ to $r = 0$. Express this in seconds for a black hole with mass measured in solar masses. Show that this maximum proper time is achieved by falling freely with $E \to 0$.

(5 pts)

2. **Hawking radiation**

a) Estimate the typical wavelength of photons radiated by a black hole of mass M and compare it with the size of the Schwarzschild radius.

b) The temperature of a sufficiently small black hole can be high enough to produce protons as component of the Hawking radiation. Estimate the required mass M of such black holes and compare their Schwarzschild radius with the size of the proton (ie. its Compton length).

(5 pts)