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Quantum Fields in Curved Spacetime

Examples VII

To hand in Monday 1st December in the examples class

1. Horizon of de Sitter space

Consider de Sitter space, i.e. an FRW metric with $a(t) = e^{Ht}$. Consider a null geodesic $x(t)$ that starts at the point $x = 0$. Compute the geodesic trajectory $x(t)$ and show that the geodesic does not reach any point further than the coordinate value $x_{max} = 1/H$.

(5 pts)

2. Mode functions in de Sitter space

By a change of variables

$$v(\eta) \equiv \sqrt{k|\eta|}f(s), \quad s \equiv \sqrt{k|\eta|}, \quad (1)$$

transform the mode equation

$$v_k'' + \left[k^2 - \left(2 - \frac{m^2}{H^2} \right) \frac{1}{\eta^2} \right] v_k = 0 \quad (2)$$

into the Bessel equation

$$s^2 \frac{d^2 f}{ds^2} + s \frac{df}{ds} + (s^2 - n^2) f = 0, \quad (3)$$

which has the general solution

$$f(s) = AJ_n(s) + BY_n(s), \quad (4)$$

where A and B are constants. Use the asymptotics of the Bessel functions $J_n(s)$, $Y_n(s)$ at large and small s to determine the asymptotics of the mode functions $v_k(\eta)$ for $k|\eta| \gg 1$ and $k|\eta| \ll 1$. (Look up the properties of the Bessel function in a handbook of mathematical functions!)

(5 pts)