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Quantum Fields in Curved Spacetime

Examples V

To hand in Monday 17th November in the examples class

1. **Energy-momentum tensor for electromagnetic fields:** Starting from the action

$$S = \tfrac{1}{4} \int d^4x \sqrt{-g}\, F^{\mu\nu} F_{\mu\nu}\,, \tag{1}$$

where $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field strength tensor, calculate the corresponding energy-momentum tensor using

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}}\,. \tag{2}$$

Moreover, calculate the component T_{00} in terms of the electric and magnetic fields $\vec{E}$ and $\vec{H}$.

(5 pts)

2. **Einstein equation:** Consider the Einstein equation

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G\, T_{\mu\nu}\,. \tag{3}$$

Invert this equation and express the Ricci curvature tensor $R_{\mu\nu}$ in terms of the energy-momentum tensor $T_{\mu\nu}$.

(2 pts)

3. **Friedmann-Robertson-Walker metric:** Consider the equation of motion for a classical scalar field ϕ in the Friedmann-Robertson-Walker spacetime $ds^2 = a^2(\eta)[d\eta^2 - dx_1{}^2 - dx_2{}^2 - dx_3{}^2]$, where η is the conformal time:

$$g^{\mu\nu} \nabla_\mu \nabla_\nu \phi + \frac{\delta V}{\delta \phi} = 0\,. \tag{4}$$

Show that this equation implies

$$\phi'' + 2\frac{a'}{a}\phi' - \Delta\phi + m^2 a^2 \phi = 0\,, \tag{5}$$

if $V = \frac{1}{2} m^2 \phi^2$. The prime denotes derivatives with respect to the conformal time and Δ is the three-dimensional Laplacian for the three space coordinates.

(3 pts)