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Quantum Fields in Curved Spacetime

Examples IV

To hand in Monday 10th November in the examples class

1. **Covariant Derivative:**

a) Discuss in detail the necessity of introducing a covariant derivative in general relativity. Compare with the case of classical electrodynamics on flat space, i.e. with a $U(1)$ gauge theory.

b) Show that for a manifold with a metric Christoffel connection, the covariant derivative ∇_μ of a vector field A^μ is given by

$$\nabla_\mu A^\mu = \frac{1}{\sqrt{|g|}} \partial_\mu \left(\sqrt{|g|} A^\mu \right) , \tag{1}$$

where g is the determinant of the metric.

(5 pts)

2. **Classical Scalar Field on Curved space:**

Consider the classical action for a conformally coupled scalar field in four space-time dimensions,

$$S = \frac{1}{2} \int d^4x \sqrt{|g|} \left(g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{1}{6} R \phi^2 \right) . \tag{2}$$

Show that this action is invariant under conformal transformations. The scalar transforms as $\delta\phi = \sigma(x)\phi$ under these transformations.

(5 pts)