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Quantum Fields in Curved Spacetime

Examples III

To hand in Monday 3rd November in the examples class

1. **Time-dependent Oscillator:**

A harmonic oscillator has a time dependent frequency $\omega(t)$ defined by

$$\omega(t) = \omega_1 \text{ for } t < 0\,, \quad \omega(t) = \omega_2 \text{ for } t > 0\,, \tag{1}$$

where ω_1, ω_2 are positive constants. The oscillator is in the ground state for all $t < 0$.

a) Determine the Bogolyubov coefficients relating the creation/annihilation operators for $t < 0$ and for $t > 0$.

b) Determine the mean occupation number and the mean energy for $t > 0$.

(5 pts)

2. **General Relativity:**

a) Using your favourite book on General Relativity, give the definitions of **metric, covariant derivative, Christoffel connection and Riemann curvature tensor**. Is the Christoffel connection a tensor? Explain.

b) Consider a vector A_α parallely transported along a small closed curve $x^\mu(s)$. Show that the change in A_α after the parallel transport can be approximately expressed as

$$\delta A_\alpha \equiv \oint \Gamma^\beta{}_{\alpha\gamma}(x) A_\beta dx^\gamma \approx \tfrac{1}{2} R^\delta{}_{\alpha\beta\gamma} A_\delta \oint x^\beta dx^\gamma\,, \tag{2}$$

where it is assumed that the area within the closed curve is very small.

Hint: Use a locally inertial coordinate system where $\Gamma^\alpha{}_{\beta\gamma} = 0$ at one point. Also, show that

$$\oint x^\alpha dx^\beta = - \oint x^\beta dx^\alpha\,. \tag{3}$$

(5 pts)