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Quantum Fields in Curved Spacetime

Examples II

To hand in Monday 27th October in the examples class

1. **Causality, Lorentz and Poincaré symmetry:**

a) Discuss the issue of causality for relativistic quantum field theory in flat Minkowski space. Explain why it is necessary to introduce fields to ensure causality in the quantized theory.

b) Using your favourite textbook, find the definitions of both Lorentz and Poincaré transformations. Which groups do they generate? Show that if a separation $x - y$ is spacelike, a continuous Lorentz transformation can take $(x - y)$ to $-(x - y)$.

c) Show that the vacuum state in a relativistic quantum field theory is invariant under Poincaré transformations.

(5 pts)

2. **Casimir effect:** Consider a massless scalar field $\phi(x,t)$ in (1+1) dimensions between conducting plates at $x = 0$ and $x = L$ with mode expansion

$$\phi(x,t) = \sqrt{\frac{2}{L}} \sum_{n=1}^{\infty} \frac{\sin \omega_n x}{\sqrt{2\omega_n}} \left[a_n^- e^{-i\omega_n t} + a_n^+ e^{i\omega_n t} \right] . \tag{1}$$

a) Show that the mode coefficients satisfy the commutation relation

$$[a_m^-, a_n^+] = \delta_{mn} . \tag{2}$$

Use the fact that for integer m, n

$$\int_0^L \mathrm{d}x \, \sin \frac{m\pi x}{L} \sin \frac{n\pi x}{L} = \int_0^L \mathrm{d}x \, \cos \frac{m\pi x}{L} \cos \frac{n\pi x}{L} = \frac{L}{2} \delta_{mn} . \tag{3}$$

b) Show that the zero-point energy per unit length is given by

$$\varepsilon_0 \equiv \langle 0|H|0\rangle = \frac{1}{2L} \sum_k \omega_k = \frac{\pi}{2L^2} \sum_{n=1}^{\infty} n . \tag{4}$$

(5 pts)