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Quantum Fields in Curved Spacetime

Examples XI

To hand in Monday 12th January in the examples class

Stable Equilibrium of black holes and heat reservoirs

In the lecture it was argued that a black hole can be in stable equilibrium with a heat reservoir only if the heat reservoir has finite energy, and therefore also finite heat capacity C_R. The following exercises discuss under which circumstances black holes can be in equilibrium with the reservoir.

(i) Find the range of heat capacities C_R of the heat reservoir for which a black hole of mass M is in a stable equilibrium with the reservoir.

Hint: In a stable equilibrium, the total entropy $S_{tot} = S_{BH} + S_R$ of the black hole and the heat reservoir is maximized, but the total energy $E_{tot} = E_{BH} + E_R$ of the system is kept constant.

(ii) Assume that the reservoir is a completely reflecting cavity of volume V filled with thermal radiation with energy $E_R = \sigma\gamma VT^4$ ($\sigma = \frac{\pi^2}{60}$ is the Stefan-Boltzmann constant in Planck units, and γ is a constant characterizing the number of degrees of freedom in the radiation fields). Determine the largest volume V of the heat reservoir for which a black hole of mass M can remain in stable equilibrium with the radiation.

Hint: Use the upper limit of the heat capacity C_R of the reservoir to determine the largest possible volume V_{max} as a function of the temperature T of the system.

(10 pts)