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Introduction to Gauge/Gravity Duality

Examples IX

To hand in Friday 13th January in the examples class

I. **Thermodynamics of the AdS Schwarzschild black hole**

Consider the following metric of a $(d+1)$ dimensional asymptotically AdS Schwarzschild black hole

$$ds^2 = \frac{L^2}{r^2}\left(f(r)d\tau^2 + \frac{dr^2}{f(r)} + dx^i dx_i\right),$$

with $f(r) = 1 - \left(\frac{r}{r_H}\right)^d$.

a) Determine the Ricci tensor $R_{\mu\nu}$ and the Ricci scalar R (It is not necessary to do a calculation, a good argument is enough)!

(2 points)

b)* Consider the (euclidean) action

$$S_E = -\frac{1}{2\kappa^2}\int d^{d+1}x\sqrt{g}\,(R - 2\Lambda) + \frac{1}{2\kappa^2}\int_{r\to 0} d^dx\sqrt{\gamma}\left(-2K + \frac{2(d-1)}{L}\right),$$

where $\Lambda = -d(d-1)/(2L^2)$ is the cosmological constant, $\gamma_{\mu\nu}$ is the induced metric on the boundary $r \to 0$, n^μ is an outward pointing normal vector to the boundary and $K = \gamma^{\mu\nu}\nabla_\mu n_\nu$ is the trace of the extrinsic curvature.

Compute the on-shell action by plugging the metric of the AdS Schwarzschild black hole into the euclidean action S_E.

[Result: the euclidean on-shell reads

$$S_E = -\frac{(4\pi)^d L^{d-1}}{2\kappa^2 d^d} V_{d-1} T^{d-1},$$

where V_{d-1} is the (spatial) volume of the corresponding field theory.] (4 points)

c) Compute the partition function

$$Z = \exp(-S_E)$$

as well as the free energy F and the entropy S

$$F = -T\log Z, \qquad S = -\frac{\partial F}{\partial T}.$$

(4 points)

*This is a very tedious exercise. Note, however, that the result is given at the end of the exercise b). Therefore you can skip exercise b) and continue with exercise c).