

Introduction to Gauge/Gravity Duality

Examples VII

To hand in Friday 13th December in the examples class

I. Saturation of unitarity bound

Consider a free scalar field with mass m in the Poincare patch of euclidean AdS_{d+1} with radius R .

a) Show that the usual bulk action

$$S_1 = -\frac{1}{2} \int d^d x dz \sqrt{g} (g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + m^2 \phi^2) ,$$

evaluated for solutions of the form $\phi(x, z) \sim z^\Delta e^{ikx}$ near $z = 0$, is finite if $\Delta > d/2$, i.e. the solution with $\Delta = \Delta_+$ (Δ_+ being the larger root of $\Delta(\Delta - d) = m^2 R^2$) are normalizable with respect to the action S_1 .

(4 points)

b) Consider the bulk action

$$S_2 = -\frac{1}{2} \int d^d x dz \sqrt{g} \phi (-\square_g + m^2) \phi .$$

Show that S_2 can be obtained by adding a boundary term to the action S_1 (hint: partial integration) and that the equations of motion are the same as for the action S_1 .

(2 points)

c) Show, that the action S_2 evaluated for solutions of the form $\phi(x, z) \sim z^\Delta e^{ikx}$ near $z = 0$ is finite if $\Delta > (d - 2)/2$. Conclude that for

$$-\frac{d^2}{4} < m^2 R^2 < -\frac{d^2}{4} + 1$$

both solutions, i.e. $\Delta = \Delta_+$ and $\Delta = \Delta_-$ are normalizable with respect to the action S_2 .

(4 points)

For more additional information see arXiv: hep-th/9905104.

II. Fefferman-Graham expansion

Consider the $(d + 1)$ -dimensional AdS metric in the form

$$ds^2 = L^2 \left(\frac{d\rho^2}{4\rho^2} + \frac{1}{\rho} g_{ij}(x, \rho) dx^i dx^j \right). \quad (1)$$

Consider a scalar field in this space with boundary expansion

$$\Phi(x, \rho) = \rho^{(d-\Delta)/2} \phi(x, \rho), \quad (2)$$

$$\phi(x, \rho) = \phi_{(0)}(x) + \rho \phi_{(2)}(x) + \rho^2 \phi_{(4)}(x) + \dots \quad (3)$$

a) Derive the equation of motion for the scalar $\phi(x, \rho)$. (2 points)

b) Using the equation of motion, show that

$$\phi_{(2)}(x) = \frac{1}{2(2\Delta - d - s)} \square_{(0)} \phi_{(0)}(x). \quad (4)$$

(3 points)