

Introduction to Gauge/Gravity Duality

Examples VII

To hand in Friday 9th December in the examples class

I. Saturation of unitarity bound

Consider a free scalar field with mass m in the Poincare patch of euclidean AdS_{d+1} with radius R .

a) Show that the usual bulk action

$$S_1 = -\frac{1}{2} \int d^d x dz \sqrt{g} (g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + m^2 \phi^2) ,$$

evaluated for solutions of the form $\phi(x, z) \sim z^\Delta e^{ikx}$ near $z = 0$, is finite if $\Delta > d/2$, i.e. the solution with $\Delta = \Delta_+$ (Δ_+ being the larger root of $\Delta(\Delta - d) = m^2 R^2$) are normalizable with respect to the action S_1 .

(4 points)

b) Consider the bulk action

$$S_2 = -\frac{1}{2} \int d^d x dz \sqrt{g} \phi (-\square_g + m^2) \phi .$$

Show that S_2 can be obtained by adding a boundary term to the action S_1 (hint: partial integration) and that the equations of motion are the same as for the action S_1 .

(2 points)

c) Show, that the action S_2 evaluated for solutions of the form $\phi(x, z) \sim z^\Delta e^{ikx}$ near $z = 0$ is finite if $\Delta > (d - 2)/2$. Conclude that for

$$-\frac{d^2}{4} < m^2 R^2 < -\frac{d^2}{4} + 1$$

both solutions, i.e. $\Delta = \Delta_+$ and $\Delta = \Delta_-$ are normalizable with respect to the action S_2 .

(4 points)

For more additional information see arXiv: hep-th/9905104.