

Introduction to Gauge/Gravity Duality

Examples V

To hand in Friday 29th November in the examples class

I. The AdS/CFT duality

a) State the precise AdS_5/CFT_4 duality in the strongest form, i.e. for general ranks N of the gauge group and for arbitrary 't Hooft coupling constants λ ! What is the strong and weak form of the duality? Which limits are taken on both sides of the duality?

Hint: Helpful relations: $g_{YM}^2 = 4\pi g_s$, $\lambda = Ng_{YM}^2$ and $L^4 = 4\pi g_s N \alpha'^2$. (3 points)

b) Show that the number of degrees of freedom per site in the d -dimensional field theory is proportional to the size of the AdS boundary, i.e. show that

$$N^2 \propto \frac{L^{d-1}}{G_N} \quad (1)$$

with L the AdS radius and G_N is the Newton constant in d dimensions.
(4 points)

c) What is the field-operator map? What are normalizable and non-normalizable modes and what is their meaning on the field theory side? (2 points)

II. Comparison of gravity and gauge theory absorption cross-sections

a) Compute the decay rate of a scalar ϕ (dilaton) into two $SU(N)$ gauge bosons using the relevant part of the worldvolume theory action of D3-branes

$$S = \int d^4x \left[-\frac{1}{4} e^{-\phi} \text{tr}(F^2) \right] \quad (2)$$

and the bulk action

$$S_{bulk} = \frac{1}{2\kappa_{10}} \int d^{10}x \sqrt{g} \left[R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \dots \right]. \quad (3)$$

This corresponds to the absorption cross-section of a dilaton by N coincident D3-branes.
(4 point)

b) The gravitational background of extremal black 3-brane is given by the metric

$$ds^2 = H(r)^{-1/2} (-dt^2 + dx_i dx^i) + H(r)^{1/2} (dr^2 + r^2 d\Omega_5^2), \quad (4)$$

with $H(r) = 1 + R^4/r^4$ and $i \in \{1, 2, 3\}$.

The absorption cross-section for a minimally coupled s-wave scalar with energy ω in this background is given by

$$\sigma_{3-brane} = \frac{\pi^4}{8} \omega^3 R^8. \quad (5)$$

Rewrite this absorption cross-section for N coincident microscopic D3-branes by using the following definition for R

$$R^4 = \frac{\kappa_{10} N}{2\pi^{5/2}}. \quad (6)$$

(1 point)

c) Compare the two cross-sections obtained in a) and b)! Why is this surprising and what is the relation to gauge-string duality? (3 points)