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Introduction to Gauge/Gravity Duality

Examples V

To hand in Friday 25th November in the examples class

I. The AdS_5/CFT_4 duality

a) State the precise AdS_5/CFT_4 duality in the strongest form, i.e. for general ranks N of the gauge group and for arbitrary 't Hooft coupling constants λ! What is the strong and weak form of the duality? Which limits are taken on both sides of the duality?

Hint: Helpful relations: $g_{YM}^2 = 4\pi g_s$, $\lambda = N g_{YM}^2$ and $R^4 = 4\pi g_s N \alpha'^2$. (3 points)

b) What is the operator–field map? What are normalizable and non–normalizable modes and what is their meaning on the field theory side? (2 points)

II. Bulk Gauge Fields in AdS/CFT

Let us consider massive gauge fields in AdS_{d+1} by the Proca action

$$S = \int\limits_{AdS} d^{d+1}x \sqrt{g} \left(\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{m^2}{2} A_\mu A^\mu \right) .$$

a) Derive the equations of motion for A_μ in the Poincaré patch of euclidean AdS_{d+1}!

Hint: The metric is given by

$$ds^2 = \frac{1}{z^2} \left(dz^2 + \delta_{\mu\nu} dx^\mu dx^\nu \right) .$$

(2 points)

b) Determine the index Δ by inserting the ansatz $A_\mu(z) = z^\Delta$ into the equations of motion! (2 points)

c) What is the scaling dimension of the corresponding current on the field theory side, which is dual to the bulk gauge field? The coupling of the current to the bulk field is given by

$$\int\limits_{\partial AdS_{d+1}} d^d x \sqrt{\gamma} A_\mu J^\mu ,$$

where $\gamma_{\mu\nu}$ is the induced metric on the conformal boundary of AdS_{d+1}. (1 point)