

## Introduction to Gauge/Gravity Duality

### Examples IV

To hand in Friday 18th November in the examples class

#### I. DBI action for a Dp-brane

The Dirac-Born-Infeld (DBI) action for a Dp-brane reads

$$S_{Dp} = -\tau_p \int d^{p+1}\zeta e^{-\Phi} \sqrt{-\det(\mathcal{P}[g]_{ab} + 2\pi\alpha' F_{ab})},$$

where  $\Phi$  is the dilaton,  $g_{\mu\nu}$  the metric of the curved spacetime and  $\mathcal{P}$  the Pullback on the worldvolume (given by coordinates  $\zeta^a$ ). Moreover,  $F_{ab}$  is the  $U(1)$  field strength tensor.  $\det$  denotes the determinant of the matrix  $\mathcal{P}[g]_{ab} + 2\pi\alpha' F_{ab}$ .

a) Simplify the action to a flat target spacetime with a vanishing dilaton. Furthermore, the Dp-brane is aligned along the coordinate axis and the embedding functions into the target spacetime (given by coordinates  $X^m$ ) vanish, i.e.  $X^a = \zeta^a$  for  $a = 0, \dots, p$  and  $X^m = 0$  for  $m = p+1, \dots, 9$ . (1 points)

b) Show that we can expand  $\det(1 + \epsilon M)$  for small  $\epsilon$  in the form

$$\det(1 + \epsilon M) = 1 + \frac{1}{2} \epsilon \operatorname{tr} M + \epsilon^2 \left( \frac{1}{8} (\operatorname{tr} M)^2 - \frac{1}{4} \operatorname{tr} (M^2) \right) + \mathcal{O}(\epsilon^3)$$

How does the expansion read for antisymmetric  $M$ .

Hint:  $\det(A) = \exp(\operatorname{Tr} \ln A)$  and expand the right side of the equation. (2 points)

c) Expand the simplified DBI action of exercise a) using the results of b) up to the first non-trivial order of the field strength tensor  $F$ . (2 points)

#### II. Near Horizon limit of M2-branes

Let us consider the near horizon limit of M2-branes. The supergravity solution of M2-branes reads

$$\begin{aligned} ds^2 &= H(r)^{-2/3} (-dt^2 + dx^2 + dy^2) + H(r)^{1/3} (dr^2 + r^2 d\Omega_7^2), \\ F_{(4)} &= dt \wedge dx \wedge dy \wedge dH^{-1}, \end{aligned}$$

where  $H(r)$  is given by

$$H(r) = 1 + \frac{L^6}{r^6}, \quad \text{where } L^6 = 32\pi^2 N l_p^6.$$

a) Take the near horizon limit  $r \rightarrow 0$  and calculate the metric and the four-form  $F_{(4)}$  in this limit. (3 points)

b) Use the following coordinate transformation  $z = \frac{L^3}{2r^2}$  and compute the metric as well as the four-form  $F_{(4)}$  in the coordinates  $(z, t, x, y, \Omega_7)$ . (2 points)