Priv.-Doz. Dr. Johanna Erdmenger
Steffen Müller

Introduction to Gauge/Gravity Duality

Examples III

To hand in Friday 11th November in the examples class

I. **Geodesics in global AdS**

We consider *global AdS* given by the coordinates (ρ, τ, Ω_i) (see also examples II) and the metric

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = L^2 \left(-\cosh^2 \rho \; d\tau^2 + d\rho^2 + \sinh^2 \rho \; d\Omega_i^2 \right) .$$

a) Find the trajectory $\tau(\rho)$ of a massless, radially-directed geodesics, starting from $\rho = \rho_0$ with proper time $\tau(\rho_0)$. What is the coordinate time for a geodesics to go from ρ_0 to the boundary and come back? What is the proper time measured by a stationary observer's clock at ρ_0 for this trajectory? Comment on this!

(2 points)

b) What is the behaviour of a massive geodesics in the radial direction of the AdS? Show that a massive geodesic never reaches the conformal boundary of AdS. Hints:

(i) The norm of the velocity $u^\mu = \frac{dx^\mu}{dT}$ is always -1, where T is the proper time along the worldline.

(ii) We know that $\cosh^2 \rho \; \frac{d\tau}{dT} = C = \text{const.}$ (Why?)

(3 points)

II. **Irreducible representations of massless SUSY multiplets**

The algebra for $\mathcal{N}$ supercharge operators is given by

$$\{Q^a_\alpha, \bar{Q}_{\dot{\beta}b}\} = -2\sigma^\mu_{\alpha\dot{\beta}} P_\mu \delta^a_b \qquad , \qquad \{Q^a_\alpha, Q^b_\beta\} = 2\epsilon_{\alpha\beta} Z^{ab} = -2\epsilon_{\alpha\beta} Z^{ba},$$

with $\alpha, \beta \in \{1, 2\}$, $a, b \in \{1, 2, ...\mathcal{N}\}$ and σ^μ are components of the four vector $(-\mathbf{1}, \sigma^i)$ of 2×2 matrices with the standard Pauli matrices σ^i.

For studying massless representations, we choose the Lorentz frame with $P^\mu = (E, 0, 0, E)$ and $E > 0$. We use the mostly minus metric.

a) Show that the SUSY algebra relation reduces to

$$\{Q^a_\alpha, \bar{Q}_{\dot{\beta}b}\} = -2(\sigma^\mu P_\mu)_{\alpha\dot{\beta}} \delta^a_b = \begin{pmatrix} 4E & 0 \\ 0 & 0 \end{pmatrix}_{\alpha\dot{\beta}} \delta^a_b .$$

(1 point)

b) Using the result in (a) and the second SUSY algebra relation given above, show that $Q_2^a = 0$ and $Z^{ab} = 0$. (1 point)

c) The remaining operators Q_1^a and $\bar{Q}_{\dot{1}a}$ with $a = 1, 2, ..., \mathcal{N}$ are used to represent field content of SUSY theories with $\mathcal{N}$ supercharges as follows:

- Q_1^a lowers helicity λ by $\frac{1}{2}$, i.e. $Q_1^a|\lambda\rangle \ = |\lambda - \frac{1}{2}\rangle$;
- $\bar{Q}_{\dot{1}a}$ raises helicity λ by $\frac{1}{2}$, i.e. $\bar{Q}_{\dot{1}a}|\lambda\rangle = |\lambda + \frac{1}{2}\rangle$.

Determine all the states of a gauge multiplet for $\mathcal{N} = 1, 2, 3, 4$ by starting from the highest helicity states $|\lambda\rangle = |1\rangle$ and applying products of Q_1^a operators on $|\lambda\rangle$ for all possible values of a.

(Hint: For $\mathcal{N} = D$, there are 2^D states in total.) (3 points)

d) Additional part: Determine all the states of a gauge multiplet for $\mathcal{N} = 1, 2, 3, 4$ by starting from the lowest helicity states $|\lambda\rangle = |-1\rangle$ and applying products of $\bar{Q}_{\dot{1}a}$ operators on $|\lambda\rangle$ for all possible values of a. What is special about the $\mathcal{N} = 4$ case compared to those of $\mathcal{N} = 1, 2, 3$ according to CPT invariance? (3 additional points)