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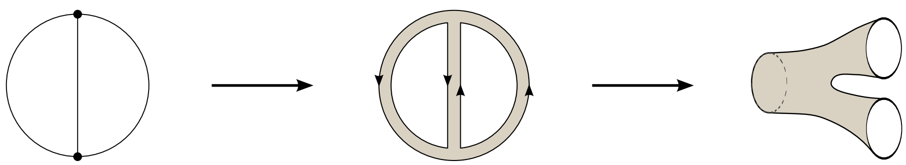
Introduction to Gauge/Gravity Duality

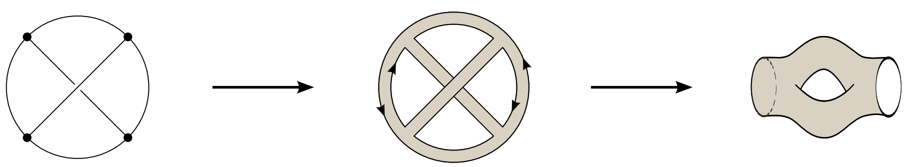
Examples II

To hand in Friday 2nd November in the examples class

I. **Large N expansion.**

Evaluate the order in N associated with the two diagrams given. (3 points)





II. **Gross-Neveu Model**

The Gross-Neveu model is defined by the Lagrangian

$$\mathcal{L} = \bar{\psi}^a i\gamma^\mu \partial_\mu \psi^a + \frac{g_0}{2N}\left(\bar{\psi}^a\psi^a\right)^2 \text{ , with } a = 1...N \tag{1}$$

in $d = 2$ spacetime dimensions. We choose the following representation for the γ^μ matrices

$$\gamma^0 = \sigma_z\,, \quad \gamma^1 = i\sigma_y \text{ and } \gamma_5 = \gamma^0\gamma^1 = \sigma_x\,, \tag{2}$$

with σ_i the standard Pauli matrices.

a) Show that this Lagrangian is symmetric under the discrete chiral transformation

$$\psi^a \to \gamma_5\psi^a\,, \; \bar{\psi}^a \to -\bar{\psi}^a\gamma_5\,. \tag{3}$$

Which terms are excluded by this symmetry? (2 points)

b) Show that this theory is renormalisable (at the level of dimensional analysis).
. (1 point)

c) Show that the Lagrangian

$$\tilde{\mathcal{L}} = \bar{\psi}^a i\gamma^\mu \partial_\mu \psi^a - \frac{N}{2g_0}\sigma^2 + \sigma\bar{\psi}^a\psi^a \tag{4}$$

is equivalent to $\mathcal{L}$ by integrating out the field $\sigma(x)$. (2 points)

d) In the large N limit the potential of this theory takes the form

$$V = \frac{N}{2g_0}\sigma^2 - \sum_{n=1}^{\infty} \frac{N}{2n} \mathrm{Tr}\left(\int \frac{\mathrm{d}^2 p}{(2\pi)^2} \left(\frac{-\not{p}\sigma}{p^2 + i\epsilon} \right)^{2n} \right) . \tag{5}$$

Explain the origin of each term. Which diagrams contribute to the second term?
. (2 points)

e) Compute the integral in equation (5) by rotating to Euclidean two-momentum p_E and integrating up to a cut off $p_E{}^2 \leq \Lambda^2$.
Result:

$$V = N\left[\frac{\sigma^2}{2g_0} + \frac{1}{4\pi}\sigma^2 \left(\ln\left(\frac{\sigma^2}{\Lambda^2} \right) - 1 \right) \right] . \tag{6}$$

Use this result to compute the renormalised coupling constant g at an arbitrary renormalisation mass M in terms of the bare coupling g_0 and the cut off:

$$\frac{1}{g} = \frac{1}{g_0} + \frac{1}{2\pi} \ln\left(\frac{M^2}{\Lambda^2} \right) + \frac{1}{\pi} . \tag{7}$$

Explain why this theory is asymptotically free. (3 points)

f) Show that the true vacuum of this theory spontaneously breaks the discrete chiral symmetry. (2 points)

References:

S. Coleman – 1/N, (1980)

D. J. Gross and A. Neveu – Dynamical symmetry breaking in asymptotically free field theories, Phys. Rev. D10:3235 (1974)