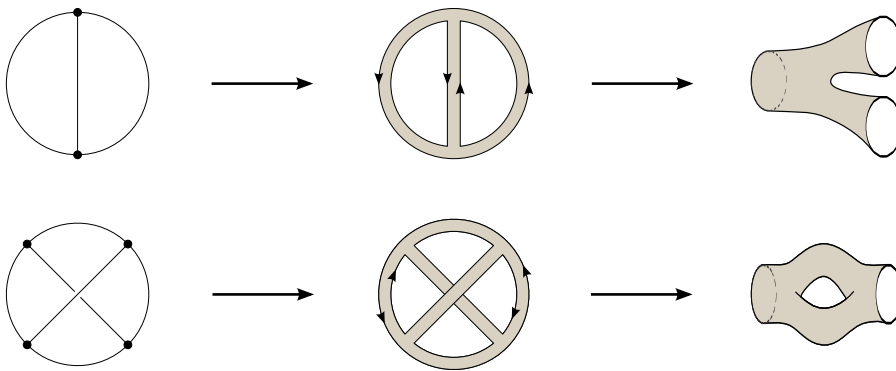Priv.-Doz. Dr. Johanna Erdmenger
Steffen Müller

## Introduction to Gauge/Gravity Duality

## Examples II

## To hand in Friday 4th November in the examples class

I. **Large $N$ expansion.**

Evaluate the order of $N$ associated with the two diagrams given.  (3 points)



II. **Coordinates of $AdS_{d+1}$.**

Lorentzian $AdS_{d+1}$ can be defined by the locus

$$-L^2 = \eta_{ab}X^a X^b = -\left(X^{d+1}\right)^2 - \left(X^0\right)^2 + \sum_{i=1}^{d}\left(X^i\right)^2 \,, \tag{1}$$

where $X \in \mathbb{R}^{2,d}$ and $ds^2 = \eta_{ab}X^a X^b$ with $\eta = \mathrm{diag}(-1,1,1,\ldots,1,-1)$. In the following we parametrize the locus (1) in different ways.

a) Draw a picture of $AdS_2$ embedded in $\mathbb{R}^{2,1}$.  (2 points)

b) The *global coordinates* $(\rho, \tau, \Omega_i)$ are defined by

$$
\begin{aligned}
X^{d+1} &= L \cosh\rho \sin\tau \,, \\
X^0 &= L \cosh\rho \cos\tau \,, \\
X^i &= L \sinh\rho \, \Omega_i \,,
\end{aligned}
$$

with $i = 1,...,d$ and $\sum_{i=1}^{d}\Omega_i^2 = 1$. Using this parametrization calculate the induced metric $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$ (where $x^\mu \in \{\rho, \tau, \Omega_i\}$) for $AdS_{d+1}$ in global coordinates.  (3 points)

c) Replace $\rho$ by $r \equiv L \sinh\rho$ and show that the metric can be written in the form

$$ds^2 = -H(r)dt^2 + H(r)^{-1}dr^2 + r^2 d\Omega_{d-1}^2,$$

where $d\Omega_{d-1}^2 = \sum_{i=1}^{d} d\Omega_i d\Omega_i.$ is the metric of the unit $(d-1)$–sphere, $S^{d-1}$.    (2 points)

d) The *Poincare patch coordinates* $(x^i, u)$ with $i = 1, ..., d$ are defined by

$$\begin{aligned} X^{d+1} + X^d &= u\,, \\ -X^{d+1} + X^d &= v\,, \\ X^i &= \frac{u}{L} x^i\,. \end{aligned}$$

Use the defining equation (1) to eliminate $v$ in terms of $u$ and $x^i$ and show that the induced metric for $(u, x^i)$ with $i = 1, ..., d$ takes the form

$$ds^2 = L^2 \frac{du^2}{u^2} + \frac{u^2}{L^2} dx^i dx_i\,.$$

Finally introduce $z = \frac{L^2}{u}$ and show that the metric is given by

$$ds^2 = \frac{L^2}{z^2} \left( dz^2 + dx^i dx_i \right)$$

Which part of the $AdS$ spacetime is not covered by these coordinates (Hint: $z$ takes only positive values (Why?)).                    (3 points)