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## Introduction to Gauge/Gravity Duality

### Examples IX

To hand in Friday 18th December in the examples class

#### I. Essay: c-Theorem

Write an essay about the c-Theorem. In particular, state the c-Theorem on the field theory side and sketch briefly the calculation on the dual gravity side. Why is the c-Theorem an important check for AdS/CFT?

Relevant literature is found in

a) c-Theorem and AdS/CFT:

D. Z. Freedman, S. S. Gubser, K. Pilch and N. P. Warner, “Renormalization group flows from holography supersymmetry and a c-theorem,” *Adv. Theor. Math. Phys.* **3** (1999) 363 [arXiv:hep-th/9904017]. Server: <http://arxiv.org/abs/hep-th/9904017>.

b) c-theorem in quantum field theory:

John Cardy: Les Houches Lecture Notes (1988): Conformal Invariance and Statistical Mechanics. May be obtained from John Cardy’s homepage:

<http://www-thphys.physics.ox.ac.uk/people/JohnCardy/>

The c-Theorem is discussed on page 51.

(5 points)

#### II. $R$ -Current Anomalies

The  $SO(6)$   $R$ -symmetry of the  $\mathcal{N} = 4SU(N_c)$  SYM theory is anomalous. This is manifest in the non-conservation of the  $R$ -current

$$\langle \partial_\mu \sqrt{g} R^\mu \rangle_{g_{\mu\nu}, V_\mu} = -\frac{a-c}{24\pi^2} R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} + \frac{5a-3c}{9\pi^2} V_{\mu\nu} \tilde{V}^{\mu\nu}, \quad (1)$$

in the presence of a metric  $g_{\mu\nu}$  and a  $R$ -current source  $V_\mu$  and its field strength tensor  $V_{\mu\nu}$ .  $a$  and  $c$  are the coefficients of the gravitational conformal anomaly. The terms  $a-c$  and  $5a-3c$  are related to triangle diagrams with chiral fermions running in the loop, with  $a-c \propto \sum_\chi r(\chi)$  and  $5a-3c \propto \sum_\chi r(\chi)^3$ , with  $r(\chi)$  the corresponding  $R$  charge.

For the undeformed  $\mathcal{N} = 4$  theory, in  $\mathcal{N} = 1$  notation, the  $R$  charge of the gaugino is  $r(\lambda) = 1$  and the one of the quarks is  $r(\psi) = -1/3$ .

Now we add a mass term to the  $\mathcal{N} = 4$  SYM theory. This term is a relevant deformation of the theory, i.e. it influences the theory in the IR. In this case we have 3 chiral fermions going around in the loop with the  $R$  charges 1,  $-1/2$  and  $-1/2$ .

- a) Draw the corresponding diagrams and explain the  $r(\chi)$  dependence. (2 points)
- b) Compute  $a - c$  and  $5a - 3c$  in the UV and IR. (2 points)
- c) Evaluate  $a_{\text{IR}}/a_{\text{UV}}$  and  $c_{\text{IR}}/c_{\text{UV}}$ ! (1 point)