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Introduction to Gauge/Gravity Duality

Examples VII

To hand in Friday 4th December in the examples class

I. **Saturation of unitarity bound**

Consider a free scalar field with mass m in the Poincare patch of euclidean AdS_{d+1} with radius R.

a) Show that the usual bulk action

$$S_1 = -\frac{1}{2} \int d^d x \, dz \sqrt{g} \left(g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + m^2 \phi^2 \right) ,$$

evaluated for solutions of the form $\phi(x,z) \sim z^\Delta e^{ikx}$ near $z = 0$, is finite if $\Delta > d/2$, i.e. the solution with $\Delta = \Delta_+$ (Δ_+ being the larger root of $\Delta(\Delta - d) = m^2 R^2$) are normalizable with respect to the action S_1.

(4 points)

b) Consider the bulk action

$$S_2 = -\frac{1}{2} \int d^d x \, dz \sqrt{g} \phi \left(-\Box_g + m^2 \right) \phi \, .$$

Show that S_2 can be obtained by adding a boundary term to the action S_1 (hint: partial integration) and that the equations of motion are the same as for the action S_1.

(2 points)

c) Show, that the action S_2 evaluated for solutions of the form $\phi(x,z) \sim z^\Delta e^{ikx}$ near $z = 0$ is finite if $\Delta > (d-2)/2$. Conclude that for

$$-\frac{d^2}{4} < m^2 R^2 < -\frac{d^2}{4} + 1$$

both solutions, i.e. $\Delta = \Delta_+$ and $\Delta = \Delta_-$ are normalizable with respect to the action S_2.

(4 points)

For more additional information see arXiv: hep-th/9905104.

II. **Fefferman-Graham expansion**

Consider the $(d+1)$-dimensional AdS metric in the form

$$ds^2 = L^2 \left(\frac{d\rho^2}{4\rho^2} + \frac{1}{\rho} g_{ij}(x,\rho) dx^i dx^j \right) . \tag{1}$$

Consider a scalar field in this space with boundary expansion

$$\Phi(x,\rho) = \rho^{(d-\Delta)/2} \phi(x,\rho) \,, \tag{2}$$

$$\phi(x,\rho) = \phi_{(0)}(x) + \rho\phi_{(2)}(x) + \rho^2 \phi_{(4)}(x) + \dots \,. \tag{3}$$

a) Derive the equation of motion for the scalar $\phi(x,\rho)$. (2 points)

b) Using the equation of motion, show that

$$\phi_{(2)}(x) = \frac{1}{2(2\Delta - d - s)} \Box_{(0)} \phi_{(0)}(x) \,. \tag{4}$$

(3 points)