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Introduction to Gauge/Gravity Duality

Examples VI

To hand in Friday 27th November in the examples class

I. **Comparison of gravity and gauge theory absorption cross-sections**

a) Compute the decay rate of a scalar ϕ (dilaton) into two $SU(N)$ gauge bosons using the relevant part of the worldvolume theory action of D3-branes

$$S = \int d^4x \left[-\frac{1}{4} e^{-\phi} \mathrm{tr}(F^2) \right] \tag{1}$$

and the bulk action

$$S_{bulk} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{g} \left[R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + ... \right] . \tag{2}$$

This corresponds to the absorption cross-section of a dilaton by N coincident D3-branes. (4 point)

b) The gravitational background of extremal black 3-brane is given by the metric

$$ds^2 = H(r)^{-1/2}(-dt^2 + dx_i dx^i) + H(r)^{1/2}(dr^2 + r^2 d\Omega_5^2) , \tag{3}$$

with $H(r) = 1 + R^4/r^4$ and $i \in \{1, 2, 3\}$.
The absorption cross-section for a minimally coupled s-wave scalar with energy ω in this background is given by

$$\sigma_{3-brane} = \frac{\pi^4}{8} \omega^3 R^8 . \tag{4}$$

Rewrite this absorption cross-section for N coincident microscopic D3-branes by using the following definition for R

$$R^4 = \frac{\kappa_{10} N}{2\pi^{5/2}} . \tag{5}$$

(1 point)

c) Compare the two cross-sections obtained in a) and b)! Why is this surprising and what is the relation to gauge-string duality? (3 points)

(please turn over)

II. **Relation between propagators in AdS**

Let us consider Euclidean AdS in the Poincaré patch.

a) Derive the equations of motion for a scalar field with mass m in Euclidean AdS_{d+1}.

(2 points)

b) Use the ansatz $\phi(z) = z^{\Delta}$ near the boundary $z \to 0$ and determine the two possible values of $\Delta_{\pm}$, where $\Delta_{+} > \Delta_{-}$.

(2 points)

c) The bulk-to-boundary propagator $K(x, z; x')$ is the solution of the equations of motion which is regular in the interior and diverges as

$$\lim_{z \to \epsilon} K(z, x; x') = \epsilon^{\Delta_{-}} \delta(x - x')$$

near the boundary, i.e. for $\epsilon \ll 1$. The bulk-to-bulk propagator is given by the solution of the equation of motion with a pointlike source term,

$$\left(\Box_{x',z'} - m^2\right) G(z', x'; z, x) = \frac{1}{\sqrt{g}} \delta(x' - x) \delta(z' - z)\,,$$

where $\Box_{x',z'}$ is the scalar Laplacian in Euclidean AdS_{d+1} acting only on x' and z'. Moreover the bulk-to-bulk propagator is regular in the interior.

Show that the bulk-to-boundary propagator $K(x, z; x')$ can be calculated from the bulk-to-bulk propagator $G(z', x'; z, x)$ by

$$K(z, x; x') = \lim_{\epsilon \to 0} \frac{\Delta_{+} - \Delta_{-}}{\epsilon^{\Delta_{+}}} G(\epsilon, x'; z, x).$$

Hint: Do not use the explicit solution for $G(z', x'; z, x)$, but Green's second identity!

(2 points)