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Introduction to Gauge/Gravity Duality

Examples V

To hand in Friday 20th November in the examples class

I. DBI action for a Dp–brane

The Dirac–Born–Infeld (DBI) action for a Dp–brane reads

$$S_{Dp} = -\tau_p \int d^{p+1}\zeta \, e^{-\Phi} \sqrt{-\det(\mathcal{P}[g]_{ab} + 2\pi\alpha' F_{ab})}\,,$$

where Φ is the dilaton, $g_{\mu\nu}$ the metric of the curved spacetime and $\mathcal{P}$ the pullback on the worldvolume (given by coordinates ζ^a). Moreover, F_{ab} is the $U(1)$ field strength tensor. det denotes the determinant of the matrix $\mathcal{P}[g]_{ab} + 2\pi\alpha' F_{ab}$.

a) Simplify the action to a flat target spacetime with a vanishing dilaton. Furthermore, take the Dp–brane to be aligned along the coordinate axis and the embedding functions into the target spacetime (given by coordinates X^m) vanish, i.e. $X^a = \zeta^a$ for $a = 0, \dots, p$ and $X^m = 0$ for $m = p, \dots, 9$. (1 point)

b) Show that we can expand $\sqrt{\det(\mathbb{1} + \epsilon M)}$ for small ϵ in the form

$$\sqrt{\det(\mathbb{1} + \epsilon M)} = 1 + \frac{1}{2}\epsilon \operatorname{tr} M + \epsilon^2 \left(\frac{1}{8} (\operatorname{tr} M)^2 - \frac{1}{4} \operatorname{tr}(M^2) \right) + \mathcal{O}(\epsilon^3)$$

What is the expansion for antisymmetric M?

Hint: $\det(A) = \exp(\operatorname{Tr} \ln A)$ and expand the right side of the equation. (2 points)

c) Expand the simplified DBI action of exercise a) using the results of b) up to the first non-trivial order of the field strength tensor F. (2 points)

II. Near-Horizon limit of M2–branes

Let us consider the near horizon limit of M2–branes. The supergravity solution of M2–branes reads

$$\begin{aligned} ds^2 &= H(r)^{-2/3} \left(-dt^2 + dx^2 + dy^2 \right) + H(r)^{1/3} \left(dr^2 + r^2 d\Omega_7 \right)\,, \\ F_{(4)} &= dt \wedge dx \wedge dy \wedge dH^{-1}\,, \end{aligned}$$

where $H(r)$ is given by

$$H(r) = 1 + \frac{L^6}{r^6}\,, \quad \text{where} \quad L^6 = 32\pi^2 N l_p^6.$$

a) Take the near horizon limit $r \to 0$ and calculate the metric and the four-form $F_{(4)}$ in this limit. (3 points)

b) Use the following coordinate transformation $z = \frac{L^3}{2r^2}$ and compute the metric as well as the four-form $F_{(4)}$ in the coordinates (z, t, x, y, Ω_7). (2 points)