

Introduction to Gauge/Gravity Duality

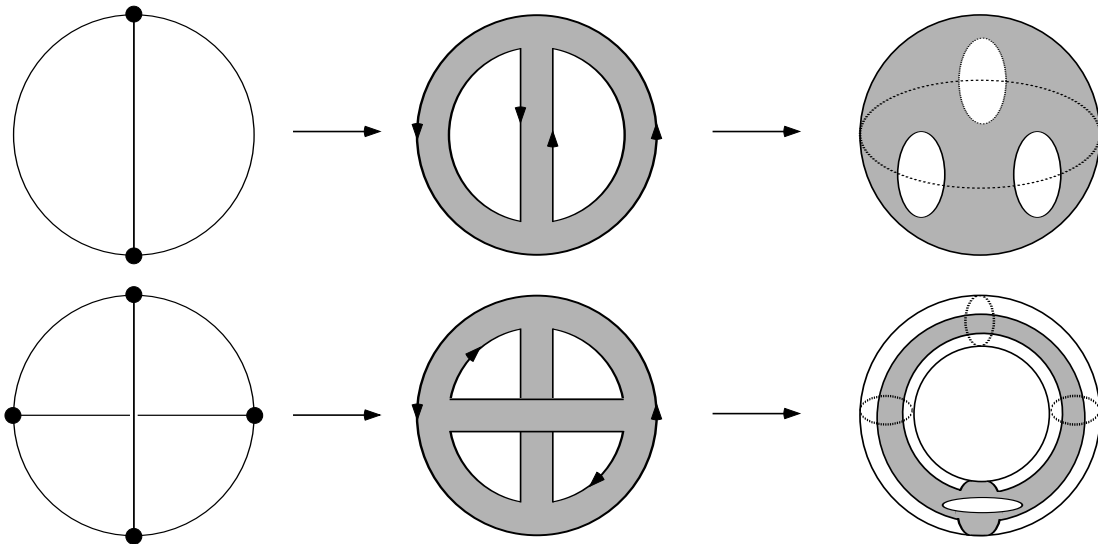
Examples II

To hand in Friday 30th October in the examples class

I. Large N expansion in gauge theories

Evaluate the order in N associated with the two diagrams given and explain the meaning of the three diagrams on each line.

(3 points)



II. Gross-Neveu Model

The Gross-Neveu model is defined by the Lagrangian

$$\mathcal{L} = \bar{\psi}^a i \gamma^\mu \partial_\mu \psi^a + \frac{g_0}{2N} (\bar{\psi}^a \psi^a)^2, \text{ with } a = 1 \dots N \quad (1)$$

in $d = 2$ spacetime dimensions. We choose the following representation for the γ^μ matrices

$$\gamma^0 = \sigma_z, \quad \gamma^1 = i\sigma_y \text{ and } \gamma_5 = \gamma^0 \gamma^1 = \sigma_x, \quad (2)$$

with σ_i the standard Pauli matrices.

a) Show that this Lagrangian is symmetric under the discrete chiral transformation

$$\psi^a \rightarrow \gamma_5 \psi^a, \quad \bar{\psi}^a \rightarrow -\bar{\psi}^a \gamma_5. \quad (3)$$

Which terms are excluded by this symmetry?

(2 points)

b) Show that this theory is renormalisable (at the level of dimensional analysis).
 . (1 point)

c) Show that the Lagrangian

$$\tilde{\mathcal{L}} = \bar{\psi}^a i\gamma^\mu \partial_\mu \psi^a - \frac{N}{2g_0} \sigma^2 + \sigma \bar{\psi}^a \psi^a \quad (4)$$

is equivalent to \mathcal{L} by integrating out the field $\sigma(x)$. (2 points)

d) In the large N limit the potential of this theory takes the form

$$V = \frac{N}{2g_0} \sigma^2 - i \sum_{n=1}^{\infty} \frac{N}{2n} \text{Tr} \left(\int \frac{d^2 p}{(2\pi)^2} \left(\frac{-\not{p}\sigma}{p^2 + i\epsilon} \right)^{2n} \right). \quad (5)$$

Explain the origin of each term. Which diagrams contribute to the second term?
 . (2 points)

e) Compute the integral in equation (5) by rotating to Euclidean two-momentum p_E and integrating up to a cut off $p_E^2 \leq \Lambda^2$.

Result:

$$V = N \left[\frac{\sigma^2}{2g_0} + \frac{1}{4\pi} \sigma^2 \left(\ln \left(\frac{\sigma^2}{\Lambda^2} \right) - 1 \right) \right]. \quad (6)$$

Use this result to compute the renormalised coupling constant g at an arbitrary renormalisation mass M in terms of the bare coupling g_0 and the cut off:

$$\frac{1}{g} = \frac{1}{g_0} + \frac{1}{2\pi} \ln \left(\frac{M^2}{\Lambda^2} \right) + \frac{1}{\pi}. \quad (7)$$

Explain why this theory is asymptotically free. (3 points)

f) Show that the true vacuum of this theory spontaneously breaks the discrete chiral symmetry. (2 points)

References:

S. Coleman – 1/N, (1980)

D. J. Gross and A. Neveu – Dynamical symmetry breaking in asymptotically free field theories, Phys. Rev. D10:3235 (1974)