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**Introduction to Gauge/Gravity Duality**

**Examples IX**

**To hand in Friday 12th December in the examples class**

## I. Essay: c-Theorem

Write an essay about the c-Theorem. In particular, state the c-Theorem on the field theory side and sketch briefly the calculation on the dual gravity side. Why is the c-Theorem an important check for AdS/CFT?

Relevant literature is found in

a) c-Theorem and AdS/CFT:

D. Z. Freedman, S. S. Gubser, K. Pilch and N. P. Warner, "Renormalization group flows from holography supersymmetry and a c-theorem," Adv. Theor. Math. Phys. 3 (1999) 363 [arXiv:hep-th/9904017]. Server: http://arxiv.org/abs/hep-th/9904017.

b) c-theorem in quantum field theory:

John Cardy: Les Houches Lecture Notes (1988): Conformal Invariance and Statistical Mechanics. May be obtained from John Cardy's homepage:
http://www-thphys.physics.ox.ac.uk/people/JohnCardy/
The c-Theorem is discussed on page 51.
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(5 points)

## II. $R$-Current Anomalies

The $SO(6)$ $R$-symmetry of the $\mathcal{N} = 4\, SU(N_c)$ SYM theory is anomalous. This is manifest in the non-conservation of the $R$-current

$$\langle \partial_\mu \sqrt{g} R^\mu \rangle_{g_{\mu\nu}, V_\mu} = -\frac{a-c}{24\pi^2} R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} + \frac{5a-3c}{9\pi^2} V_{\mu\nu} \tilde{V}^{\mu\nu} \,, \tag{1}$$

in the presence of a metric $g_{\mu\nu}$ and a $R$-current source $V_\mu$ and its field strength tensor $V_{\mu\nu}$. $a$ and $c$ are the coefficients of the gravitational conformal anomaly. The terms $a-c$ and $5a-3c$ are related to triangle diagrams with chiral fermions running in the loop, with $a - c \propto \sum_\chi r(\chi)$ and $5a - 3c \propto \sum_\chi r(\chi)^3$, with $r(\chi)$ the corresponding $R$ charge.

For the undeformed $\mathcal{N} = 4$ theory, in $\mathcal{N} = 1$ notation, the $R$ charge of the gaugino is $r(\lambda) = 1$ and the one of the quarks is $r(\psi) = -1/3$.

Now we add a mass term to the $\mathcal{N} = 4$ SYM theory. This term is a relevant deformation of the theory, i.e. it influences the theory in the IR. In this case we have 3 chiral fermions going around in the loop with the $R$ charges 1, $-1/2$ and $-1/2$.

a) Draw the corresponding diagrams and explain the $r(\chi)$ dependence. (2 points)

b) Compute $a - c$ and $5a - 3c$ in the UV and IR. (2 points)

c) Evaluate $a_{\mathrm{IR}}/a_{\mathrm{UV}}$ and $c_{\mathrm{IR}}/c_{\mathrm{UV}}$! (1 point)