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Introduction to Gauge/Gravity Duality

Examples VIII

To hand in Friday 5th December in the examples class

I. **Fefferman-Graham expansion**

Consider the $(d+1)$-dimensional AdS metric in the form

$$ds^2 = L^2 \left(\frac{d\rho^2}{4\rho^2} + \frac{1}{\rho} g_{ij}(x,\rho) dx^i dx^j \right) . \tag{1}$$

Consider a scalar field in this space with boundary expansion

$$\Phi(x,\rho) = \rho^{(d-\Delta)/2} \phi(x,\rho) , \tag{2}$$
$$\phi(x,\rho) = \phi_{(0)}(x) + \rho \phi_{(2)}(x) + \rho^2 \phi_{(4)}(x) + \dots . \tag{3}$$

a) Derive the equation of motion for the scalar $\phi(x,\rho)$. (2 points)

b) Using the equation of motion, show that

$$\phi_{(2)}(x) = \frac{1}{2(2\Delta - d - s)} \Box_{(0)} \phi_{(0)}(x) . \tag{4}$$

(3 points)

II. **Penrose-Brown-Henneaux Transformation**

Show that, when transforming the metric

$$ds^2 = L^2 \frac{d\rho^2}{4\rho^2} + \frac{1}{\rho} g_{\mu\nu} dx^\mu dx^\nu \tag{5}$$

according to the Penrose-Brown-Henneaux diffeomorphism

$$\rho = \rho'(1 - 2\sigma(x')) \qquad x^\mu = (x')^\mu + a^\mu(x', \rho') \tag{6}$$

and demanding that

$$g'_{55} = g_{55} \;\text{ and }\; g'_{\mu 5} = g_{\mu 5} , \tag{7}$$

one obtains

$$\partial_5 a^\mu = \frac{L^2}{2} g^{\mu\nu} \partial_\nu \sigma \tag{8}$$

and

$$g_{\mu\nu} \to g_{\mu\nu} + 2\sigma \left(1 - \rho \frac{\partial}{\partial \rho} \right) g_{\mu\nu} + \nabla_\mu a_\nu + \nabla_\mu a_\nu . \tag{9}$$

Note that the index 5 stands for the ρ direction. Explain why this transformation induces a Weyl transformation at the boundary. 5 points