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Introduction to Gauge/Gravity Duality

Examples VII

To hand in Friday 28th November in the examples class

I. **Saturation of unitarity bound**

Consider a free scalar field with mass m in the Poincaré patch of Euclidean AdS_{d+1} with radius R.

a) Show that the usual bulk action

$$S_1 = -\frac{1}{2}\int d^d x\, dz \sqrt{g}\left(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + m^2\phi^2\right)\,,$$

evaluated for solutions of the form $\phi(x,z) \sim z^\Delta e^{ikx}$ near $z = 0$, is finite if $\Delta > d/2$, i.e. the solution with $\Delta = \Delta_+$ (Δ_+ being the larger root of $\Delta(\Delta - d) = m^2R^2$) are normalizable with respect to the action S_1.

(4 points)

b) Consider the bulk action

$$S_2 = -\frac{1}{2}\int d^d x\, dz \sqrt{g}\phi\left(-\Box_g + m^2\right)\phi\,.$$

Show that S_2 can be obtained by adding a boundary term to the action S_1 (hint: partial integration) and that the equations of motion are the same as for the action S_1.

(2 points)

c) Show, that the action S_2 evaluated for solutions of the form $\phi(x,z) \sim z^\Delta e^{ikx}$ near $z = 0$ is finite if $\Delta > (d-2)/2$. Conclude that for

$$-\frac{d^2}{4} < m^2R^2 < -\frac{d^2}{4} + 1$$

both solutions, i.e. $\Delta = \Delta_+$ and $\Delta = \Delta_-$ are normalizable with respect to the action S_2.

(4 points)

For more additional information see arXiv: hep-th/9905104.