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Introduction to Gauge/Gravity Duality

Examples VI

To hand in Friday 21st November in the examples class

I. Comparison of gravity and gauge theory absorption cross-sections

a) Compute the decay rate of a scalar ϕ (dilaton) into two $SU(N)$ gauge bosons using the relevant part of the worldvolume theory action of D3-branes

$$S = \int d^4x \left[-\frac{1}{4} e^{-\phi} \text{tr}(F^2) \right] \quad (1)$$

and the bulk action

$$S_{bulk} = \frac{1}{2\kappa_{10}} \int d^{10}x \sqrt{g} \left[R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \dots \right]. \quad (2)$$

This corresponds to the absorption cross-section of a dilaton by N coincident D3-branes. (4 point)

b) The gravitational background of extremal black 3-brane is given by the metric

$$ds^2 = H(r)^{-1/2} (-dt^2 + dx_i dx^i) + H(r)^{1/2} (dr^2 + r^2 d\Omega_5^2), \quad (3)$$

with $H(r) = 1 + R^4/r^4$ and $i \in \{1, 2, 3\}$.

The absorption cross-section for a minimally coupled s-wave scalar with energy ω in this background is given by

$$\sigma_{3-brane} = \frac{\pi^4}{8} \omega^3 R^8. \quad (4)$$

Rewrite this absorption cross-section for N coincident microscopic D3-branes by using the following definition for R

$$R^4 = \frac{\kappa_{10} N}{2\pi^{5/2}}. \quad (5)$$

(1 point)

c) Compare the two cross-sections obtained in a) and b)! Why is this surprising and what is the relation to gauge-string duality? (3 points)

II. Relation between propagators in AdS

Let us consider Euclidean AdS in the Poincaré patch.

a) Derive the equations of motion for a scalar field with mass m in Euclidean AdS_{d+1} .

(2 points)

b) Use the ansatz $\phi(z) = z^\Delta$ near the boundary $z \rightarrow 0$ and determine the two possible values of Δ_\pm , where $\Delta_+ > \Delta_-$.

(2 points)

c) The bulk-to-boundary propagator $K(x, z; x')$ is the solution of the equations of motion which is regular in the interior and diverges as

$$\lim_{z \rightarrow \epsilon} K(z, x; x') = \epsilon^{\Delta_-} \delta(x - x')$$

near the boundary, i.e. for $\epsilon \ll 1$. The bulk-to-bulk propagator is given by the solution of the equation of motion with a pointlike source term,

$$(\square_{x,z} - m^2) G(z, x; z', x') = \frac{1}{\sqrt{g}} \delta(x - x') \delta(z - z'),$$

where $\square_{x,z}$ is the scalar Laplacian in Euclidean AdS_{d+1} acting only on x and z . Moreover the bulk-to-bulk propagator is regular in the interior.

Show that the bulk-to-boundary propagator $K(x, z; x')$ can be calculated from the bulk-to-bulk propagator $G(z, x; z', x')$ by

$$K(z, x; x') = \lim_{z' \rightarrow \epsilon} \frac{\Delta_+ - \Delta_-}{\epsilon^\Delta} G(z, x; z', x').$$

Hint: Do not use the explicit solution for $G(z, x; z', x')$, but Green's second identity!

(2 points)