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Introduction to Gauge/Gravity Duality

Examples V

To hand in Friday 14th November in the examples class

I. The AdS/CFT duality

a) State the precise AdS_5/CFT_4 duality in the strongest form, i.e. for general ranks N of the gauge group and for arbitrary 't Hooft coupling constants λ! What is the strong and weak form of the duality? Which limits are taken on both sides of the duality?

Hint: Helpful relations: $g_{YM}^2 = 4\pi g_s$, $\lambda = N g_{YM}^2$ and $L^4 = 4\pi g_s N \alpha'^2$. (3 points)

b) Show that the number of degrees of freedom per site in the d-dimensional field theory is proportional to the size of the AdS boundary, i.e. show that

$$N^2 \propto \frac{L^{d-1}}{G_N} \tag{1}$$

with L the AdS radius and G_N is the Newton constant in d dimensions.
(4 points)

c) What is the field-operator map? What are normalizable and non–normalizable modes and what is their meaning on the field theory side? (2 points)

II. Near-Horizon limit of M2–branes

Let us consider the near horizon limit of M2–branes. The supergravity solution of M2–branes reads

$$\begin{aligned} ds^2 &= H(r)^{-2/3}\left(-dt^2 + dx^2 + dy^2\right) + H(r)^{1/3}\left(dr^2 + r^2 d\Omega_7\right) , \\ F_{(4)} &= dt \wedge dx \wedge dy \wedge dH^{-1} , \end{aligned}$$

where $H(r)$ is given by

$$H(r) = 1 + \frac{L^6}{r^6} , \quad \text{where} \quad L^6 = 32\pi^2 N l_p^6.$$

a) Take the near horizon limit $r \to 0$ and calculate the metric and the four-form $F_{(4)}$ in this limit. (3 points)

b) Use the following coordinate transformation $z = \frac{L^3}{2r^2}$ and compute the metric as well as the four-form $F_{(4)}$ in the coordinates (z, t, x, y, Ω_7). (2 points)