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Introduction to Gauge/Gravity Duality

Examples IV

To hand in Friday 7th November in the examples class

I. **DBI action for a Dp–brane**

The Dirac–Born–Infeld (DBI) action for a Dp–brane reads

$$S_{Dp} = -\tau_p \int d^{p+1}\zeta \, e^{-\Phi} \sqrt{-\det(\mathcal{P}[g]_{ab} + 2\pi\alpha' F_{ab})}\,,$$

where Φ is the dilaton, $g_{\mu\nu}$ the metric of the curved spacetime and $\mathcal{P}$ the pullback on the worldvolume (given by coordinates ζ^a). Moreover, F_{ab} is the $U(1)$ field strength tensor. det denotes the determinant of the matrix $\mathcal{P}[g]_{ab} + 2\pi\alpha' F_{ab}$.

a) Simplify the action to a flat target spacetime with a vanishing dilaton. Furthermore, take the Dp–brane to be aligned along the coordinate axis and the embedding functions into the target spacetime (given by coordinates X^m) vanish, i.e. $X^a = \zeta^a$ for $a = 0, \dots, p$ and $X^m = 0$ for $m = p, \dots, 9$. (1 point)

b) Show that we can expand $\sqrt{\det(\mathbb{1} + \epsilon M)}$ for small ϵ in the form

$$\sqrt{\det(\mathbb{1} + \epsilon M)} = 1 + \frac{1}{2}\epsilon \,\mathrm{tr}\, M + \epsilon^2 \left(\frac{1}{8}(\mathrm{tr}\, M)^2 - \frac{1}{4}\mathrm{tr}\,(M^2)\right) + \mathcal{O}(\epsilon^3)$$

What is the expansion for antisymmetric M?

Hint: $\det(A) = \exp(\mathrm{Tr}\ln A)$ and expand the right side of the equation. (2 points)

c) Expand the simplified DBI action of exercise a) using the results of b) up to the first non-trivial order of the field strength tensor F. (2 points)

II. **Curvature of AdS and Cosmological constant**

Let us consider AdS_{d+1} in the *Poincaré patch* given by the coordinates $(z, t, \vec{x})$ and the metric

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = \frac{L^2}{z^2}\left(dz^2 - dt^2 + d\vec{x}^2\right)\,,$$

where $\vec{x}$ are the $d-1$ spatial dimensions.

a) Calculate $g^{\mu\nu}$, the Levi-Civita connection $\Gamma^\mu_{\nu\rho}$, the Riemann tensor $R^\mu{}_{\nu\rho\sigma}$ as well as the Ricci tensor $R_{\mu\nu}$ and the Ricci scalar R.

(3 points)

b) Show that AdS_{d+1} solves the vacuum Einstein field equations $G_{\mu\nu} + \Lambda g_{\mu\nu} = 0$, where Λ is the cosmological constant and $G_{\mu\nu} = R_{\mu\nu} - \frac{R}{2}g_{\mu\nu}$ is the Einstein tensor. Determine the value of the cosmological constant Λ!

(2 points)