

## Introduction to Gauge/Gravity Duality

### Examples XI

To hand in Friday 16th January in the examples class

#### I. Shear viscosity over entropy density

a) To determine its correlation functions, the energy-momentum tensor  $T^{\mu\nu}$  has to be coupled to a source, the metric  $g_{\mu\nu}$ . In this exercise we consider the following small perturbations  $h_{\mu\nu}$  around the flat metric

$$\begin{aligned} g_{ij}(t, \vec{x}) &= \delta_{ij} + h_{ij}(t), \quad h_{ij} \ll 1 \\ g_{00}(t, \vec{x}) &= -1, \quad g_{0i}(t, \vec{x}) = 0, \end{aligned} \quad (1)$$

with  $i, j$  being spatial indices.

The dissipative part of the energy-momentum tensor  $T^{\mu\nu}$  in curved space-time is

$$\sigma^{\mu\nu} = P^{\mu\alpha} P^{\nu\beta} \left[ \eta (\nabla_\alpha u_\beta + \nabla_\beta u_\alpha) + \left( \zeta - \frac{2}{3} \eta \right) g_{\alpha\beta} \nabla \cdot u \right], \quad (2)$$

with the local fluid velocity  $u^\mu = (1, 0, 0, 0)$  and the projector  $P^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$ , which projects onto the hyperplane transverse to the fluid velocity.

Compute  $\sigma^{xy}$  to leading order in  $h_{ij}$  by expanding the explicit expressions for the covariant derivatives in (2). Relate  $\sigma^{xy}$  to the retarded Green's function  $G_{xy,xy}^R(\omega, \vec{0})$ . State Kubo's formula for the shear viscosity  $\eta$ . (5 points)

b) Consider the black-brane solution of type IIB supergravity in  $AdS_5 \times S^5$

$$ds^2 = \frac{(\pi T L)^2}{u} (-f(u) dt^2 + dx^2) + \frac{L^2}{4u^2 f(u)} du^2 + L^2 d\Omega_5^2, \quad (3)$$

with  $f(u) = 1 - u^2$ ,  $T$  the temperature and  $L$  the  $AdS_5$  radius. The linearized equation of motion of the metric fluctuation  $h_{xy}$  above this background is of the form  $\partial_\mu (g^{\mu\nu} \partial_\nu \phi) = 0$ , with  $\phi = h^x_y$ .

Show that this equation of motion corresponds to the following equation in momentum space ( $' = \partial_u$ )

$$\phi'' - \frac{1+u^2}{uf} \phi' + \frac{\tilde{\omega}^2 - \tilde{k}^2 f}{uf^2} \phi = 0, \quad (4)$$

with  $\tilde{\omega} = \omega/(2\pi T)$  and  $\tilde{k} = k/(2\pi T)$ .

Using the recipe by Son and Starinets (see literature list below), we obtain the following retarded Green's function of energy-momentum tensor  $T_{xy}$ ,

$$G_{xy,xy}^R(\omega, 0) = -\frac{\pi^2 N^2 T^4}{4} i\tilde{\omega}. \quad (5)$$

Compute the ratio of the shear viscosity  $\eta$  and the entropy density  $s$  (For the entropy, note last weeks exercise sheet!). (5 points)

*Recommended literature:*

D. T. Son and A. O. Starinets, “Viscosity, Black Holes, and Quantum Field Theory,” *Ann. Rev. Nucl. Part. Sci.* **57** (2007) 95 [arXiv:0704.0240 [hep-th]].

P. Kovtun, D. T. Son and A. O. Starinets, “Viscosity in strongly interacting quantum field theories from black hole physics,” *Phys. Rev. Lett.* **94** (2005) 111601 [hep-th/0405231].

D. T. Son and A. O. Starinets, “Minkowski-space correlators in AdS/CFT correspondence: recipe and applications,” *JHEP* **0209** (2002) 042 [hep-th/0205051].